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Amplitude Modulated Line-Based Halftoning

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Abstract

We present a new approach for line-based halftoning of images, where banks of waveforms are modulated by the gray levels of the rendered image. We present examples that employ sinusoidal and triangular waveforms, and highlight some applications, including stylized rendering, halftoning in line-based devices, and education.

1. Introduction

Halftoning is the process of rendering a continuous-tone image in a monochrome (single tone) device. The primary goal of halftoning is to make the rendered image look as close as possible to the original when seen from enough distance. The paradigm, as Pnueli and Bruckstein [PB96] describe, is that "the local density of black elements should be proportional to the local grayness of the original image" . Stylized halftoning adds a requirement that a closeup reveals unexpected detailed. Conversely, a distant view surprisingly reveals an image [Ahm14]. In what we call line-based halftoning, an image is rendered with lines instead of pixels, and the Pnueli and Bruckstein condition is realized by controlling the density of the lines in different parts of the rendered image.

An early idea for line-based halftoning, proposed by Schroeder [Sch83] and elaborated by Pnueli and Bruckstein [PB96], uses contours of a "potential field" influenced by the gray levels of the image, as though grayness represents impedance to the propagation of an electromagnetic wavefront. A more recent idea, proposed by Bosch and Herman [BH04] and elaborated by Kaplan and Bosch [KB05], employs the Traveling Salesman Problem (TSP) to connect dots of a dithered image, as though these dots are cities in a TSP tour. Inspired by these ideas, we asked if there are other concepts in the scientific literature that can be employed for line-based halftoning. We found a good candidate in the Amplitude Modulation (AM) concept from Telecommunications, as discussed below.

2. Halftoning via Amplitude Modulation

When a sinusoidal signal is simulated and plotted, the signal energy is directly proportional to the amount of ink required to plot it. Suppose that time is mapped to the horizontal (x) axis of the plotting surface, and that a full cycle of the signal is taken as one unit in the plot. For the unmodulated "carrier", each unit would contain

(a)
(b)
(c)

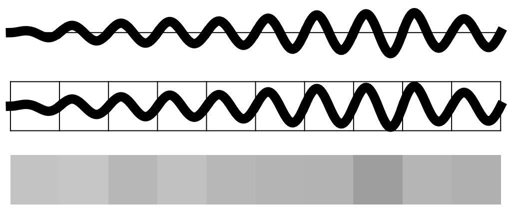

Figure 1: *Diffusing the ink in a plot of AM signal. (a) The plotted signal. (b) One-cycle-wide pixels for rasterization. (c) The resulting pixel colors.*

the same amount of ink. When the carrier is amplitude-modulated, however, we obtain longer arcs (more ink) where the modulating signal is stronger. If we use one-cycle-wide pixels to rasterize the plot, the grayness of the resulting pixels would be proportional to the energy of the modulating signal. See Figure 1.

Now we reverse the process: we use a row of pixels to modulate a sinusoidal wave, then we plot the modulated signal. To obtain a smooth waveform, the discrete pixels should be gradient-smoothed, and the centers of the pixels, where the modulating signal has non-continuous first derivatives, should be aligned with peaks or zero-crossings of the carrier.

AM line-based halftoning is obtained by modulating banks of sinusoidal waves by scan lines of the image. The peak (carrier plus signal) amplitude is adjusted so that the waves do not interfere in adjacent rows. A pixel may be set to span any whole number of cycles. The line length of a sinusoidal cycle is not linearly proportional to the amplitude, but it is not difficult to build a look-up table to convert gray levels to amplitudes that produce proportional arc-lengths of the modulated cycles. See Figure 2.

A simpler and more efficient alternative is to use triangular waves (Figure 3(a)). Compared to sinusoidal waves, they admit

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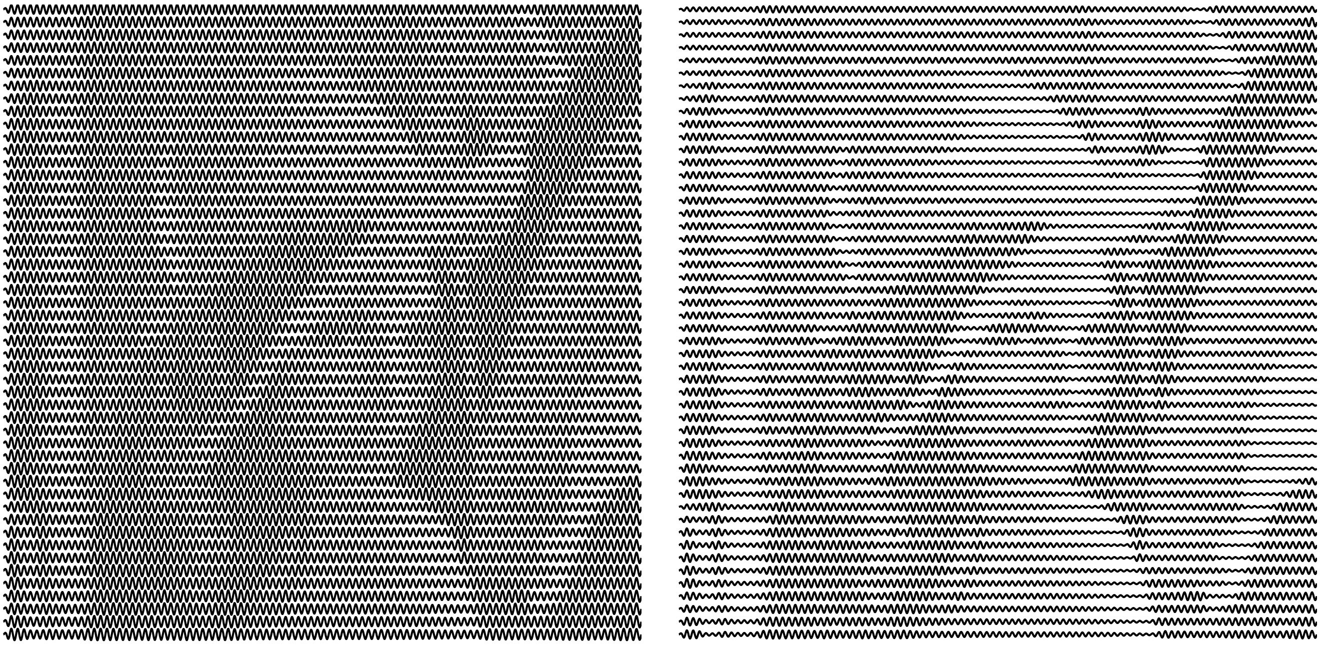


(a) (b)

Figure 2: *AM line-based halftonings, using (a) a carrier amplitude equal to the peak signal amplitude (some rows overlap), and (b) a zero-energy (suppressed) carrier.*

doubled image resolutions (only one peak-per-pixel), and they render faster (no curves), admitting real-time rendering of video.

In contrast to many line-based approaches (e.g. TSP Art), our approach is *loss-less*, in the sense that it does not "transcode" the image: it is simply a rendering algorithm for individual pixels. The rendered wavelet for each gray level could be computed precisely, so there is no need for the tone correction described in [Ahm15]. On the downside, there is some loss of contrast in the process, because a white pixel is mapped to a tone of gray.

3. Applications

3.1. Stylized Rendering

A closeup shows wavy lines, and the underlying image emerges as the viewer's eye moves farther away. It gives the illusion that the underlying image is rendered behind the wavy lines, not by the wavy lines themselves. Waveform, amplitudes (carrier and signal), and frequency (cycles per pixel) represent design parameters.

3.2. Education

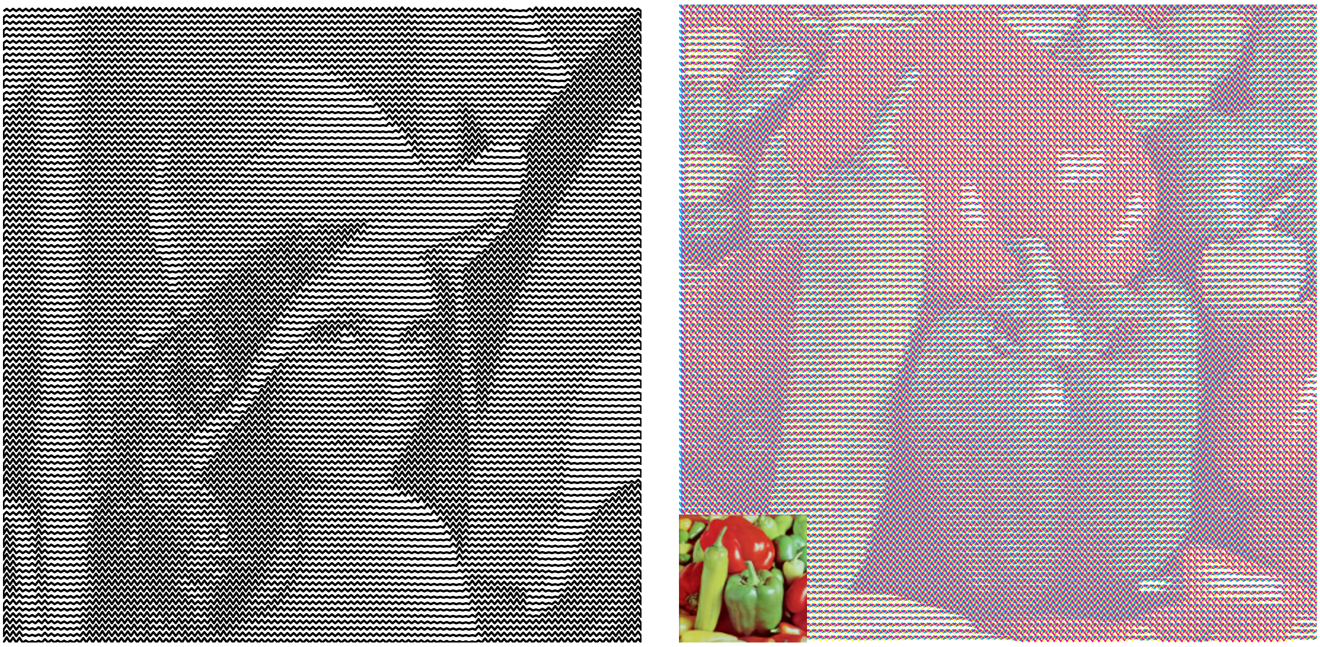
Exploiting many analogies, a whole lecture on telecommunications, for students or for non-technical audience, could be held in front of an AM-halftoned image of, say, Marconi. The viewer's eye represents the whole receiver (demodulator and decoder). For example, in radio, the carrier energy poses a burden on the receiver's circuits; analogously, it overlays a gray taint on the halftoned image that burdens our eyes trying to rule out the "DC" level of gray. This problem can be reduced by using a zero-energy (suppressed) carrier (Figure 2(b)). Next, the carrier tends to interfere with the signal when their frequencies are relatively close; analogous to the interference of the plotted curves with the perceived image. Improved performance is obtained when the carrier frequency is much higher than the signal frequency (= many cycles per pixel), but it comes at a cost in the bandwidth of the overall system (= a larger area is required to plot a legible image).

3.3. Functional Halftoning

AM line-based halftoning is well-suited for Computer Numeric Control (CNC) devices and egg-bots, since these favor analog (line-based) over digital (raster-based) halftoning.

3.4. Embroidery

The most sound application is *photo-realistic* embroidery, where we are not aware of a suitable alternative. The triangular waveform readily matches the zig-zag stitching style implemented in many commercial sewing machines. All that needed is to vary the size of sideway movements of the needle. Further, consecutive rows can be traversed in a serpentine fashion so that the whole piece can be embroidered with a single continuous yarn. It should also be possible to embroider colored halftonings (Figure 3(b)) using three or four yarns.



(a) (b)

Figure 3: *AM line-based halftonings with a triangular waveform: (a) Lena, monochrome. (b) Peppers, CMY.*

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