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A user study on quantisation thresholds of triangle meshes

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Abstract

We present the results of a user study on estimating a quantisation threshold above which the quantised triangle mesh is perceived as indistinguishable from its unquantised original. The design of the experiment and the analysis of the results focus on the comparison between two different quantisation methods: rounding, in which all bits above the threshold are put to zero; and dithering, in which all bits above the threshold are randomised. The results show that dithered meshes require more bits per vertex coordinate in order to reach the indistinguishability threshold, and while the difference between the two methods is small, around one bit per vertex coordinate, it is nevertheless statistically significant.

Categories and Subject Descriptors (according to ACM CCS): Computer Graphics [Computing methodologies]: Graphics systems and interfaces—Perception

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1. Introduction

Triangle meshes is the ubiquitous shape representation for 3D graphics and visualisation applications. In their simplest form, they consist of a set of vertices, which are points in $\mathbf{R}^3$ connected between them by triangular faces. The encoding of the vertex coordinates most often makes use of 32-bit floats, however, the use of fixed-point with less than 32 bits per vertex coordinate is also common, especially when we want the triangle mesh in a compressed form. While strictly speaking geometry encoded at any finite precision, including 32-bit floats, is quantised, here following a widely accepted convention we refer to the process of transformation from 32-bit floats to fixed-point arithmetic as *quantisation*, to the resulted mesh as *quantised* and to the original mesh as *unquantised*.

The effect of the quantisation on the visual quality of the mesh naturally depends on the *quantisation level*, that is, the number of bits per vertex coordinate. While it is well-known that coarse quantisations often result to meshes of low visual quality, to the best of our knowledge there is no systematic study aiming at finding the minimum number of bits per vertex coordinate that are required for a quantised mesh that will be visually indistinguishable from the unquantised. While there could be several possible explanations for the lack of study of this *quantisation threshold*, we note as a prominent one that the threshold seems to depend on several of the mesh characteristics in conjunction with the rendering algorithm used and that, generally, it should be considered as application dependent. A classic example where a quantisation level must be chosen outside the context of a specific visual application the testing and evaluation of mesh compression algorithms. In early seminal papers such as such as [TG98], the quantisation levels range from 8 to 10 bits per vertex, while in some of the more recent approaches surveyed in [MLDH15], the standard quantisation level seem to be 16 bits per vertex coordinate. In [IIGS05], general, not necessarily triangle meshes were tested at quantisation levels ranging from 12 to 16 bits.

The experimental study of quantisation thresholds in this paper focuses on the comparison between two different quantisation methods. The first is *rounding*, which sets all the bits above the quantisation level to zero. The second method is *dithering*, where all bits above the quantisation level are considered as having a random value. While the simplicity of rounding makes it the most commonly used quantisation method, dithering has the advantage that the randomised bits could represent encoded information in applications such as high capacity steganography [YPI13]. Fig. 1 shows an example of rounding and dithering at 8 bits per vertex coordinate.

The findings of the experiment are summarised as follows:

- Dithering has a higher threshold than rounding, that is, with dithering we need more bits per vertex coordinate to make the quantised model indistinguishable from the unquantised. The increase is small, around one bit per vertex coordinate in average in our experiments, but nevertheless statistically significant. To the best of our knowledge, this is the first paper establishing such a result.
- As expected, the characteristics of the mesh model affect the quantisation threshold. Regarding the type of characteristics affecting most the quantisation threshold, the first indications we have from our experiment suggest that the size of the model is more important than smoothness. In particular, larger models

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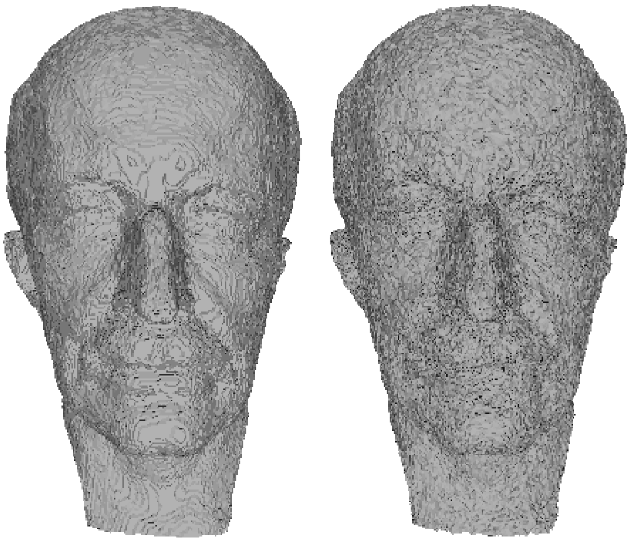


Figure 1: ***Left:*** *Rounding at 8 bits per vertex coordinate.* ***Right:*** *Dithering at 8 bits per vertex coordinate.*

with many triangles and thus more detail require, as expected, more bits per vertex coordinate.

The main limitation of our approach is that we use only one rendering method. Moreover, by opting for the interface of experiment to be interactive the renderings presented to the participants were of low quality, while on the other hand though it should be noted that our rendering setting, essentially Phong shading, is often met in real-world applications. The second limitation is that the set of models we used was limited to three models in total, even though their characteristics were very diverse. Overall, while we think that the comparison of the two quantisations methods was fair and broad enough to have limited only threats to the validity of the main finding that dithering has higher quantisation thresholds, the results regarding the effect of the mesh characteristics on the thresholds should be considered preliminary, and of course, the actual values of the thresholds computed in each case should be treated with caution as application depended.

The rest of the paper is organised as follows. In Section 2 the related work is reviewed. In 3 we describe the experimental setup, in 4 we present the outcome of the experiment and we briefly conclude in 5.

2. Background

Quantisation techniques are most often studied in the context of signal theory [GG12]. According to an extensive survey of the technique in [GN98], rounding, which is historically the oldest example of quantisation and was first analysed in [She97] for estimating densities by histograms. Dithering was introduced in [Rob62] for improving the visual quality of a digitally encoded image by removing the visual artifacts caused by coarse quantisations of the grayscale range.

2.1. Perception

Subjective experiments have been employed by various researchers studying 3D model visual quality degradation under common mesh manipulation processes such as lossy compression [WFM01], or watermarking [CGEB07]. More recent work utilises large databases containing meshes that have undergone a variety of distortions including compression, lossy tranmission and noise addition [SSFM09,Lav09], while in [TWC15] dynamic meshes are considered. The types of mesh distortions studied in those papers are not as simple and natural as the vertex coordinate quantisations of our case, and the ultimate aim there is not just a comparison between two specific distortions, but rather the development and validation of metrics of visual mesh quality which can then be computed automatically.

We are not aware of any systematic experimental comparison between the quantisation thresholds of rounding and dithering. It is of course well-known that rounding creates blocky artifacts, which could be easily detected by the human visual system. On the other hand, dithering causes high frequency noise which human observers are also sensitive to. With 2D images, blockiness in the form of averaging of pixel values over a given area, is known to decrease visual recognition performance [HJ73]. Similarly to the 3D model case, such blocky artifacts may be the result of certain lossy image compression algorithms. Although dithering in the form of added noise also degrades performance in many visual tasks [PF99], it can actually improve performance when added to a blocky 2D image: the added noise disrupts the high-frequency edge structure of the blocky image, making it easier to recognise [MBR83].

The perceptual effect of quantisation of a 3D model is, of course, more complex. The stimulus for the human observer is not the quantised model itself, but a 2D image that is a result of a rendering process. As such, the perceptual effects of quantisation depend on the rendering algorithm and, eventually, how blocky the result appears depends at least partly on how good a job the rendering algorithm does in smoothing out the quantisation effects. On the other hand, the noise introduced in dithering might itself be highly visible to the observer, possibly making the quantised version perceptually even more dissimilar from the original.

Given that blockiness resulting from vertex coordinate rounding and high frequency noise introduced by dithering are both causes of visual degradation, it was difficult formulate a firm hypothesis prior to the execution of the experiment on how the quantisation thresholds of the two methods compare. Instead, we expected statistically non-significant differences as the most probable outcome of the experiment and lower dithering thresholds as the second most probable outcome, given the cues we had from the literature on possible visual improvement of images through dithering. While the eventual outcome of the experiment was the opposite, i.e.,lower thresholds for rounding, it should be noted that we did not compare the general visual quality of the two quantisation methods but something rather more specific, i.e. the indistinguishability thresholds.

3. Experiment

For a given quantisation level l, the x coordinates of the mesh vertices were rounded by scaling them to the interval $[0, 2^l - 1]$ and rounding to the nearest integer, that is

$$x \to \text{round}(\frac{x - \min_x}{\max_x - \min_x}(2^l - 1)) \tag{1}$$

where $\max_x$ and $\min_x$ are the minimum and maximum value of the x coordinate, respectively. The rounded x coordinates were then multiplied by $\max_x - \min_x$ to retain the proportions of the original mesh. The y and z coordinates were treated similarly.

To produced the dithered quantised meshes, we added a uniformly random variable from the interval [0,1] to the rounded integer coordinates of Eq. 1, before rescaling by $\max_x - \min_x$. Notice that there were many other possibilities regarding both the amount of added noise and its type, e.g., blue or pink noise instead of white. Here, the choice was informed by our aim to study the visual effect of least significant bit watermarking on triangle mesh coordinates.

In each trial of the experiment the participant was presented with two meshes, one unquantised at the left hand side of the screen and a quantised one at the right hand side of the screen. The participant had to decide if the two meshes were different or not by giving a Yes/No answer to the question *Do the two meshes look the same?*. The interface of the experiment was interactive, allowing the user to use the mouse to grab any of the two meshes and rotate them, or zoom in and out of them. All implementation was done in Matlab and a screenshot of the interface is shown in Figure 2.

The three meshes, chosen primarily for their large variation in size, are shown in Figure 3. The smallest was the *Cube* with 766 vertices, the *Eight* with 15K vertices was chosen as mid-sized and the *Max-Plank* model with 100K vertices as large. We also note that there is significant variation in the natural characteristics of the models: the Cube is a CAD model with sharp features, the Eight is an analytic model that is very smooth and has non-trivial topology, while the Max-Planck is a natural model which contains both smooth areas and sharp features.

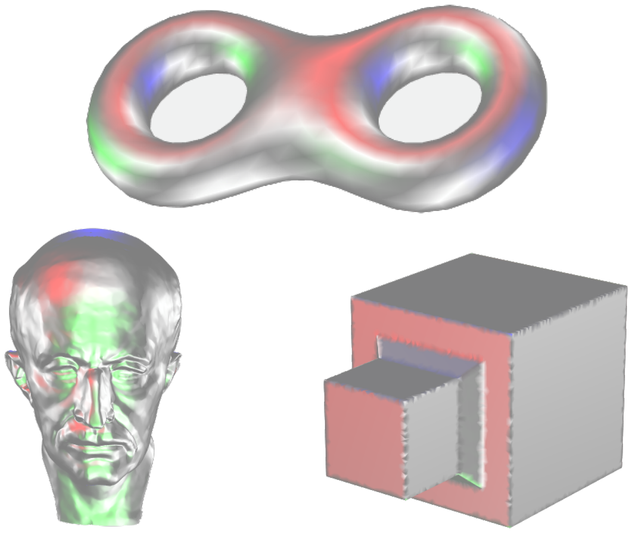


Figure 3: *The models used in the experiments were the Eight, the Max-Planck and the Cube.*

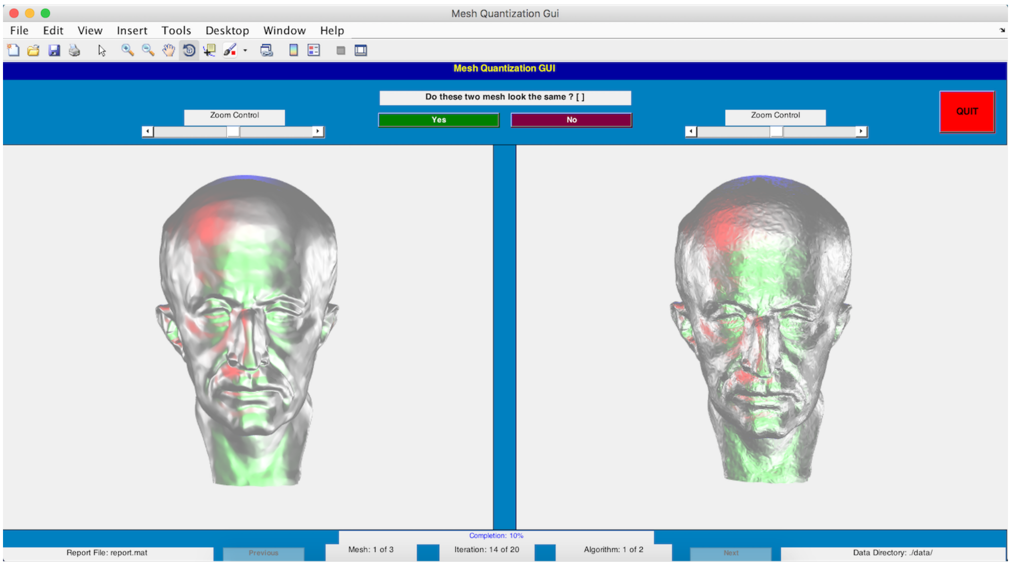
The three models and the two quantisation methods created a 2-dimensional space of six in total conditions. For each condition the participant was presented with 20 trials meaning 120 trials in total. The order in which meshes were presented was fixed as Eight, Cube and Max-Planck, while the order in which the quantisation algorithms were presented was random. After a Yes answer, meaning that the participant was perceiving the two models as looking the same, meaning in turn that the quantisation level was on or above the threshold, quantisation level of the next trial was decremented by one. After a No answer the level of quantisation was incremented by one. As it has been established in the literature [Cor62, RTR70], in this type of experiments it is useful to start a staircase series of trials as near to the actual threshold as possible. Therefore, we established rough estimates of the thresholds by running a pilot and then the set of 20 trials for each condition was starting at these estimated thresholds. For example, for the Maxc-Planck model the initial threshold estimated by the pilot was 12 bits per vertex coordinate for either of the two quantisation methods. Figure reffig:maxSeries shows a series of renderings for the dithered quantised Max-Planck model around the initially estimated threshold.

The pilot was conducted in November 2016 at Durham University while the main experiment was conducted in January 2017 with a convenience sample of 21 students from Qassim University, Saudi Arabia. Ethical clearance for the experiment was obtained from Durham University. At the beginning of the experiment the participants were signing consent forms and were given a brief oral introduction to the purpose of the experiment. Next, they were presented with a pre-trial using a mesh that was different from the three meshes of the main experiment before, finally, being presented with the main experiment. There were no time limits for any single trial, or for the whole experiment, and no timings were recorded, however, all participants completed the experiment in around 30 minutes. Data from twenty one participants in total were collected and analysed, but as we discuss in Section 4, data from one participant were excluded as outliers.

4. Results

For each participant and for each of the six conditions of the experiment we compute a point estimate of the quantisation threshold, which is not necessarily an integer number, as follows. From the corresponding set of 20 Yes/No trials we exclude the first five. The exclusion of a number of initial trials is for allowing the staircase to reach the threshold and is recommended in [Cor62]. The estimate of the quantisation threshold is then computed as the average of the first two peaks and the first two valleys.

Next, we screened the results for possible exclusions of outliers. This step is highly recommended, not only in subjective but also in physical experiments [CBS*15]. In a user study, screening for outliers can lead to the exclusion of participants from all or parts of the analysis, or to the exclusion of results associated with parts of the experimental dataset [PW03]. In our case, participant number 16 was found to be above the average quantisation threshold by more than two standard deviations for four out of the six conditions and was excluded from any further analysis. We believe that this participant systematically overestimated the threshold by a high margin due to a misunderstanding of the instructions. There were



Do these two mesh look the same ? [ ]

Completion: 10%

Report File: report.mat

Mesh: 1 of 3

Iteration: 14 of 20

Algorithm: 1 of 2

Data Directory: ./data/

Figure 2: *The interactive interface of the experiment.*

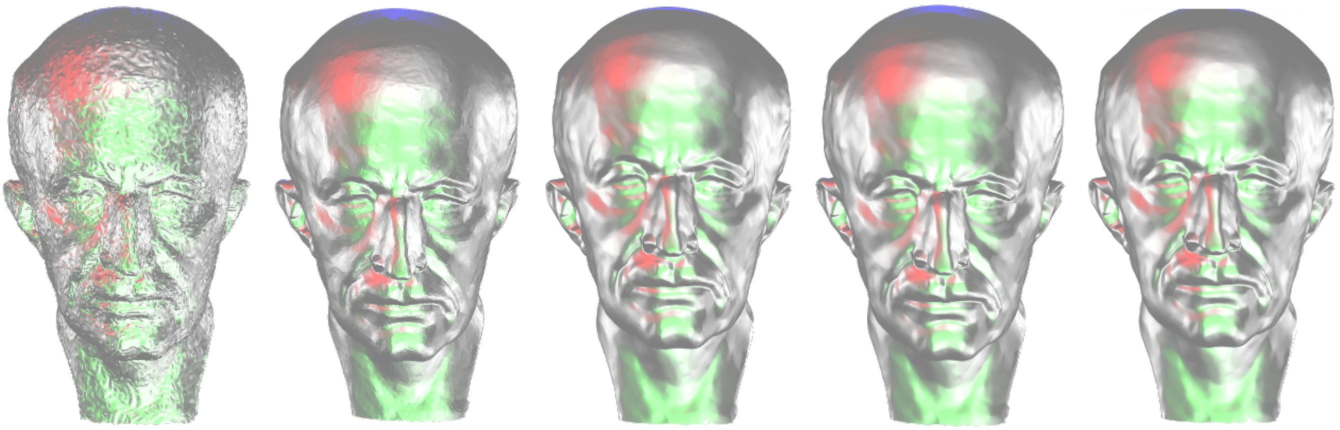


Figure 4: ***From left to right:*** *The Max-Planck model at dithered quantisation levels of 8,10,12,14 and 16 bit per vertex coordinate.*

three more participants that were outside the ±2 standard deviation zone in one of the six conditions, but they were not excluded. We note that here we did not follow the empirical recommendations of ITU [BT598] protocol for participant exclusion, firstly because their recommendation does not explicitly cover the format of our experiment, i.e. a Yes/No staircase, and secondly because it seems to be very strict when the data are not deemed normally distributed in which case the outlier zone is $\pm\sqrt{20}$ standard deviations.

	S-W p-value	skewness
Cube Trunc.	.006	1.299
Cube Dith.	.003	1.389
Eight Trunc.	.376	.032
Eight Dith.	.018	-1.182
Max Planck Trunc.	.539	-.222
Max Planck Dith.	.001	1.914

Table 1: *The results of the Shapiro-Wilks normality test and the skewness of the distributions*

4.1. Normality tests

Table 1 shows the results of Shapiro-Wilks normality test for each condition. We notice that in four out of the six cases the data are classified as non-normal and the non-normality can be the result of either positive or negative skewness.

Figure 5 shows frequency histograms for the Truncated Cube and the Dithered Max-Planck models. In the case of the Cube, which has a low number of vertices and thus low quantisation threshold, the non-normality can be attributed to a naturally one-sided distribution of the observed thresholds. That is, the left tail of the distribution is very short because it was quite unlikely that a participant would underestimate considerably the threshold. On the other hand, for higher quantisation thresholds as in the case of the Max-Planck model, the high skewness value seems to be the result of outliers.

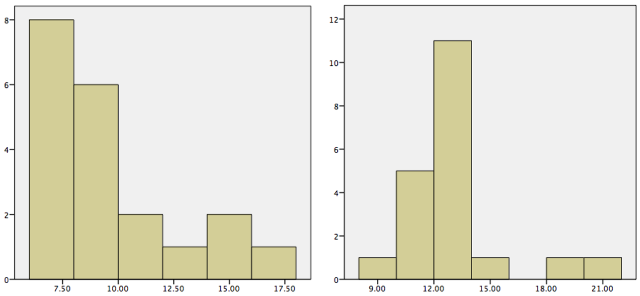


Figure 5: ***Left:*** *The frequency histogram of the estimated thresholds for the Cube with truncation.* ***Right:*** *The frequency histogram of the estimated thresholds for the Max-Planck with dithering.*

4.2. ANOVA test and post-hoc analysis

Since ANOVA tests are considered robust under non-normality conditions, we proceeded with a 2-way ANOVA test. The quantisation method is significant with $p = 0.045$ and $F = 4.094$, while the mesh is significant with $p < 0.001$ and $F = 11.248$. Figure 6 shows the averages for each condition of the experiment and we notice that there is a small but consistent across the three meshes difference between the average thresholds of the two quantisation methods.

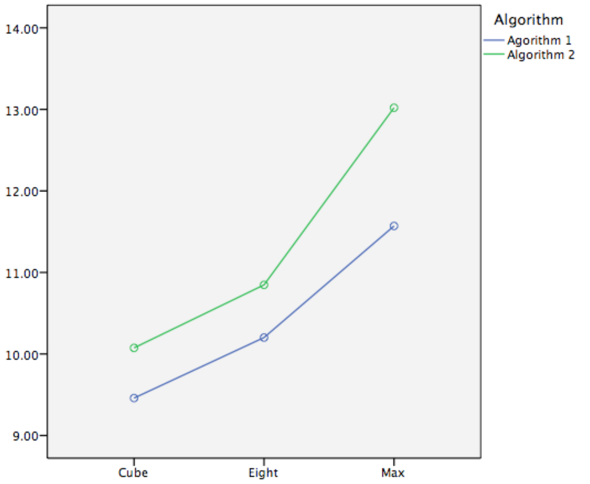


Figure 6: *The means for each mesh for truncation (top line) and dithering (bottom line).*

Finally, in a post-hoc analysis of the results we performed pairwise comparisons between the three meshes after collapsing the quantisation method variable. Figure 6 shows boxplots for the three meshes. The difference between Cube and Eight was not statistically significant with a $p = .506$ value for the Bonferroni correction test. On the other hand, Max-Planck was significantly different from Cube and Eight with $p < 0.001$ and $p = 0.005$ for the corresponding Bonferroni correction tests.

While the focus of the experiment was on the comparison between the two quantisation methods and thus, it was not designed to answer questions regarding the effect of mesh characteristics on the quantisation threshold, we note that the results indicate that the size of the mesh is the most important factor in determining the quantisation threshold.

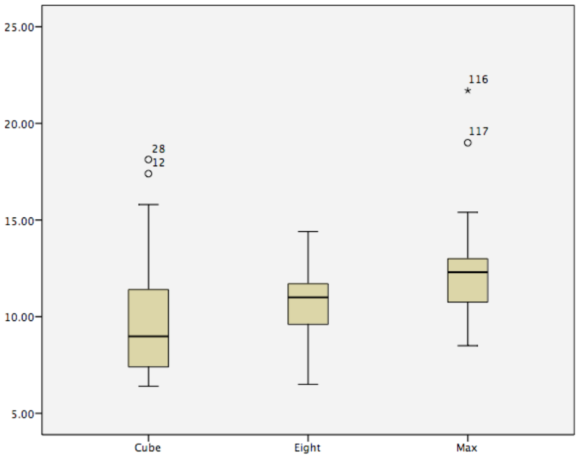


Figure 7: *The boxplots of the meshes after collapsing the quantisation method variable.*

5. Conclusions

We presented an experimental study of the quantisation threshold of triangle mesh vertices, above which a quantised mesh becomes visually indistinguishable from the original unquantised. The focus of our study was the comparison between two quantisation methods, rounding and dithering, and our main finding was that dithering has a higher quantisation threshold than rounding. While that result does not contradict any prior findings of the existing literature, we note that it could not have been easily predicted before the execution of actual experiment since, in the particular setting of 3D model quantisation, it was not known a priori whether blockiness or high frequency noise would prove to be perceptually stronger.

In the future we plan to work on the more complex and thus more challenging problem of studying the relationship between quantisation thresholds, geometric properties of the mesh and properties of the rendering algorithms used. Such a study would require higher dimensional experiment and perhaps more subtle experimental designs too. In particular, we plan to use the maximum likelihood difference scaling method which has been proven to be a powerful approach to similar problems [MY03, CMCK07].

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