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Rendering Transparent Materials with a Complex Refractive Index: Semi-conductor and Conductor Thin Layers

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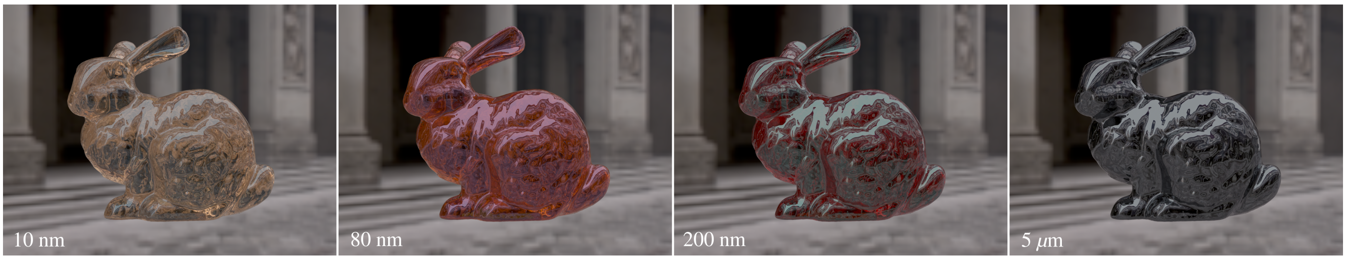

Figure 1: *A sample object made of transparent glass and covered with a thin hematite (Fe_2O_3) layer. The layer thickness is increasing from left to right. Taking into account attenuation and multiple bounces inside the hematite layer produces a changing appearance depending on the thickness.*

Abstract

During physical simulation of light transport, we separate materials between conductors and dielectrics. The former have a complex refractive index and are treated as opaque, the latter a real one and are treated as transparent. However, thin layers with a complex refractive index can become transparent if their thickness is small compared to the extinction coefficient. This happens with thin metallic layers, but also with many pigments that are semi-conductors: their extinction coefficient (the imaginary part of their refractive index) is close to zero for part of the visible spectrum. Spectral effects inside these thin layers (attenuation and interference) result in dramatic color changes.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Computing methodologies— Reflectance modeling; Rendering

1. Introduction

For material representation, we usually separate between *conductors*, or opaque materials, with a complex refractive index, and *dielectrics*, or transparent materials, with a real refractive index. Physically, attenuation of electromagnetic waves inside the conductor is related to the imaginary part of the refractive index, through an exponential falloff. The larger the imaginary part is, the faster the intensity decreases. For most practical uses, a solid block of conductor can be treated as opaque.

However, with thin metallic layers (for example less than 30 nm for gold), a significant amount of incoming light is transmitted through the layer. Many pigments, such as hematite (Fe_2O_3), Cadmium sulfide (CdS), vermillon (HgS), are semi-conductors: their extinction coefficient is null or close to 0 for part of the visible spectrum. Part of the visible light is transmitted, even for relatively large layers. The relationship between transmission and attenuation plays an important part in the pigments color (see Figure 1).

In this paper, we present a spectral model for conductors and semi-conductors, taking into account both attenuation and interference through the layer.

2. Previous works

2.1. Refractive index and color

Akenine-Möller et al. [AMHH*18] contains a detailed description of the connection between refractive index and reflectance properties of materials. They separate between dielectrics (transparent)

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and conductors (opaque). From a physics point of view, Born and Wolf [BW13] explains the connection between the physical properties of materials and the way they reflect and transmit electromagnetic waves.

2.2. Thin layers

When light enters a thin layer coating on a larger object, multiple reflections inside the layer produce subtle effects, due to difference in optical path length between rays. Hirayama et al. [HKYM01, HKY*01] present an algorithm for rendering these effects with multiple transparent layers. The paper contains pictures with thin metallic layers but only gives formulas for dielectrics.

Belcour and Barla [BB17] provide a fast method for rendering these interference effects in thin transparent layers. Their key idea is to project the effect onto three well chosen basis functions, providing real-time rendering. The method is efficient, but limited to transparent materials, with a refractive index that does not depend on the wavelength.

3. Background

Materials are defined by their refractive index, η. Dielectric materials are transparent to light and have a real refractive index, corresponding to the ratio between the speed of light inside the material and the speed of light in the vacuum (see Figure 2a). Conductors have a complex refractive index with a non-null imaginary part: $\eta = n + ik$ (see Figure 2b). The refractive index depends on the wavelength and this variation is responsible for the color of the material. The imaginary part of the refractive index is called the *extinction coefficient* and is connected to the attenuation of electromagnetic waves inside the material.

3.1. Transmission

The amount of light being reflected R on a specular interface, and the amount of light being transmitted T inside the material are expressed by reflection and transmission coefficients, often called Fresnel coefficients. Depending on the polarization of the incident light :

$$r_s = \frac{\eta_1 cos(\theta_i) - \eta_2 cos(\theta_t)}{\eta_1 cos(\theta_i) + \eta_2 cos(\theta_t)} \qquad r_p = \frac{\eta_2 cos(\theta_i) - \eta_1 cos(\theta_t)}{\eta_2 cos(\theta_i) + \eta_1 cos(\theta_t)}$$
$$t_s = \frac{2\eta_1 cos(\theta_i)}{\eta_1 cos(\theta_i) + \eta_2 cos(\theta_t)} \qquad t_p = \frac{2\eta_1 cos(\theta_i)}{\eta_2 cos(\theta_i) + \eta_1 cos(\theta_t)}$$

For unpolarized light, the reflection coefficient is the average of the perpendicular and parallel polarized coefficients:

$$R = \frac{|r_s|^2 + |r_p|^2}{2} \tag{1}$$
$$T = \frac{\eta_2 cos(\theta_t)}{\eta_1 cos(\theta_i)} \frac{|t_s|^2 + |t_p|^2}{2} \tag{2}$$

$|t|^2$ being the ratio between transmitted and incident *intensities*, and T being defined as the ratio between transmitted and incident *flux*, the factor $\eta_2 \cos\theta_t / \eta_1 \cos\theta_i$ ensures the conversion between these two physical quantities.
For energy conservation, $R + T = 1$.

4. Thin conductor layers

Under normal circumstances, conductors are opaque. But for thin enough layers, light can be transmitted, even through a conductor. Practical examples include glass panels covered with layers of aluminium or copper alloys in modern buildings. The layer has to be extremely thin. For standard metals most of the light is blocked after about 100 nm. With these dimensions we need to take into account the wave nature of light and interference effects.

Consider an object with refractive index η_3, covered with a thin conductor layer with refractive index $\eta_2 = n_2 + ik_2$, of thickness d, inside a transparent medium of refractive index η_1 (see Figure 3). Taking into account multiple bounces inside the layer and the optical path difference, we can compute the reflectance and transmittance coefficients.

The complex transmission and reflection coefficients at each interface come from Equations 3.1; we write r_{12}, t_{12}, r_{23} and t_{23} the reflection and transmission coefficients at the 1-2 and 2-3 interfaces respectively. For each outgoing direction, we get the sum of multiple contributions: light that was reflected on the top interface (R_1), plus light that was transmitted at the top interface, reflected on the bottom interface and transmitted again at the top interface (R_2), and so on with multiple bounces inside the layer (R_3, R_4).

There is an optical path difference δ between each of these contributions, corresponding to the distance travelled by the light. This optical path difference results in a phase difference ϕ between the different rays, and thus into interference effects. It corresponds to the difference between the optical path length light travelled on the first ray R_1, going from I_1 to H and the optical path length on the second ray R_2, going from I_1 to I_2:

$$\delta = (I_1 J_1 I_2) - (I_1 H) \tag{3}$$
$$= \frac{2\eta_2 d}{\cos\theta_2} - 2\eta_2 d \frac{\sin^2\theta_2}{\cos\theta_2} \tag{4}$$
$$= 2\eta_2 d \cos\theta_2 \tag{5}$$

This complex optical path difference δ results in a complex phase difference between the rays $\phi = \frac{2\pi}{\lambda}\delta$.

We sum the contributions from all the reflected rays R_i, with $i-1$ bounces inside the conductor layer; R_1 is the light reflected on the top interface, R_2 is the light leaving after one bounce inside the layer and so on. For each ray R_i, its amplitude is the amplitude of the incoming light multiplied by the attenuation coefficient r_i:

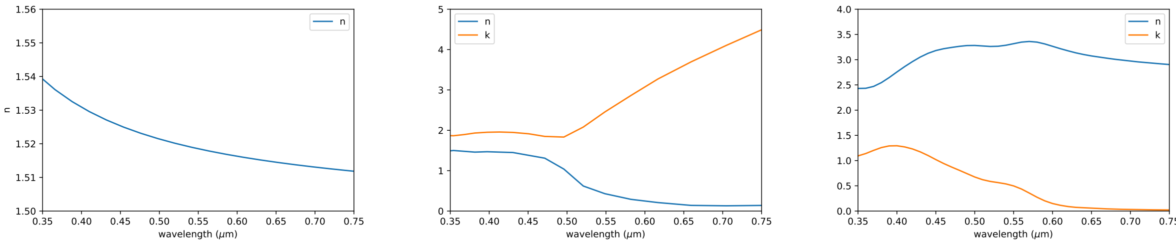
$$r_0 = r_{12} \tag{6}$$
$$r_1 = t_{12} r_{23} t_{21} e^{i\phi} \tag{7}$$
$$r_n = t_{12} r_{23} (r_{23} r_{21})^n t_{21} e^{in\phi} \tag{8}$$

Since $r_{21} = -r_{12}$ and $t_{12}t_{21} = 1 - r_{12}^2$, we have:

$$r_n = (-1)^n \left(1 - r_{12}^2\right) r_{23}^{n+1} r_{12}^n e^{in\phi} \tag{9}$$

The total contribution is the sum of contributions from all the rays:

$$r = \sum_{n=0}^{\infty} r_n = r_{23}\left(1 - r_{12}^2\right) \sum_{n=0}^{\infty} \left(-r_{23} r_{12} e^{i\phi}\right)^n \tag{10}$$



(a) *Glass (BK7)* **(b)** *Gold (conductor)* **(c)** *Hematite (semi-conductor)*

Figure 2: *Refractive index for several materials, as a function of wavelength*

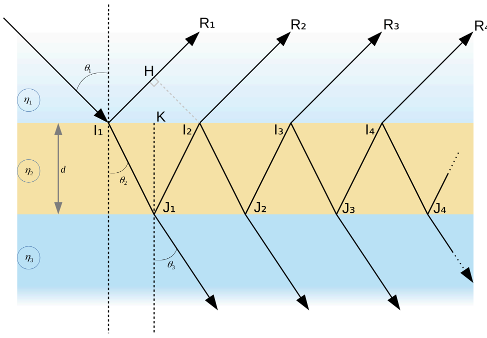


Figure 3: *Multiple bounces inside a thin layer.*

After simplification, we get:

$$r = \frac{r_{12} + r_{23}e^{i\phi}}{1 + r_{12}r_{23}e^{i\phi}} \tag{11}$$

Similarly for transmission, we have:

$$t = \frac{t_{12}t_{23}e^{i\frac{\phi}{2}}}{1 + r_{12}r_{23}e^{i\phi}} \tag{12}$$

This computation is valid for each component of polarized light, s and p, taken individually and produces a complex number. To get the real reflected coefficient R, we need to sum the square modulus of the polarized coefficients (see Equation 1): $R = (|r_s|^2 + |r_p|^2)/2$.

To get these real coefficients R and T, we need the modulus and phase of r_{12} and r_{23}, and the real and imaginary part of the phase difference ϕ:

$$r_{12} = \rho_{12}e^{i\varphi_{12}} \tag{13}$$
$$r_{23} = \rho_{23}e^{i\varphi_{23}} \tag{14}$$
$$\phi = \phi_R + i\phi_I \tag{15}$$

With these, we get $|r|^2 = r\bar{r}$:

$$|r|^2 = \frac{\rho_{12}^2 e^{\phi_I} + \rho_{23}^2 e^{-\phi_I} + 2\rho_{12}\rho_{23}\cos(\varphi_{23} - \varphi_{12} + \phi_R)}{e^{\phi_I} + \rho_{12}^2\rho_{23}^2 e^{-\phi_I} + 2\rho_{12}\rho_{23}\cos(\varphi_{23} + \varphi_{12} + \phi_R)} \tag{16}$$

This expression is numerically unstable for large values of the imaginary part of the phase difference ϕ_I, due to the exponential term. We divide numerator and denominator by e^{ϕ_I} to get a stable version:

$$|r|^2 = \frac{\rho_{12}^2 + \rho_{23}^2 e^{-2\phi_I} + 2\rho_{12}\rho_{23}e^{-\phi_I}\cos(\varphi_{23} - \varphi_{12} + \phi_R)}{1 + \rho_{12}^2\rho_{23}^2 e^{-2\phi_I} + 2\rho_{12}\rho_{23}e^{-\phi_I}\cos(\varphi_{23} + \varphi_{12} + \phi_R)} \tag{17}$$

This formula is valid for both polarizations, r_s and r_p, substituting the respective coefficients, r_{12s} and r_{23s} or r_{12p} and r_{23p} as needed. We then get the real reflection coefficient R with: $R = (|r_s|^2 + |r_p|^2)/2$.

Similarly, we write the modulus and phase of the transmission coefficients t_{12} and t_{23}:

$$t_{12} = \tau_{12}e^{i\psi_{12}} \tag{18}$$
$$t_{23} = \tau_{23}e^{i\psi_{23}} \tag{19}$$

and get the complex transmission coefficient across the layer:

$$|t|^2 = \frac{\tau_{12}^2\tau_{23}^2 e^{-\phi_I}}{1 + \rho_{12}^2\rho_{23}^2 e^{-2\phi_I} + 2\rho_{12}\rho_{23}e^{-\phi_I}\cos(\varphi_{12} + \varphi_{23} + \phi_R)} \tag{20}$$

This expression is valid for each component of the polarized light, s and p. To get the real transmssion coefficient T, we average the contributions of the s and p components:

$$T = \frac{\eta_3\cos\theta_3}{\eta_1\cos\theta_1}\frac{|t_s|^2 + |t_p|^2}{2} \tag{21}$$

As the layer thickness increases, d increases and ϕ_I increases with it. In the limit case, we get $|r|^2 = \rho_{12}^2$ and $|t|^2 = 0$.

5. Results

Figure 4 and 1 show an object made of glass, covered with a layer of varying thickness. The change in color and appearance of the object is related to the layer thickness. The observed color is the sum of the color that is reflected at the surface of the object and the color that is transmitted through the object. The thicker the object is, the less the transmitted light contributes to the observed color. With gold, above 100 nm, the object is almost opaque. With Cadmium sulfide, the object remains partially transparent even at 1 mm. For pigments, such as hematite and Cadmium sulfide, the layer thickness has a visible impact on the material hue.

For certain material combinations, interference effects result in

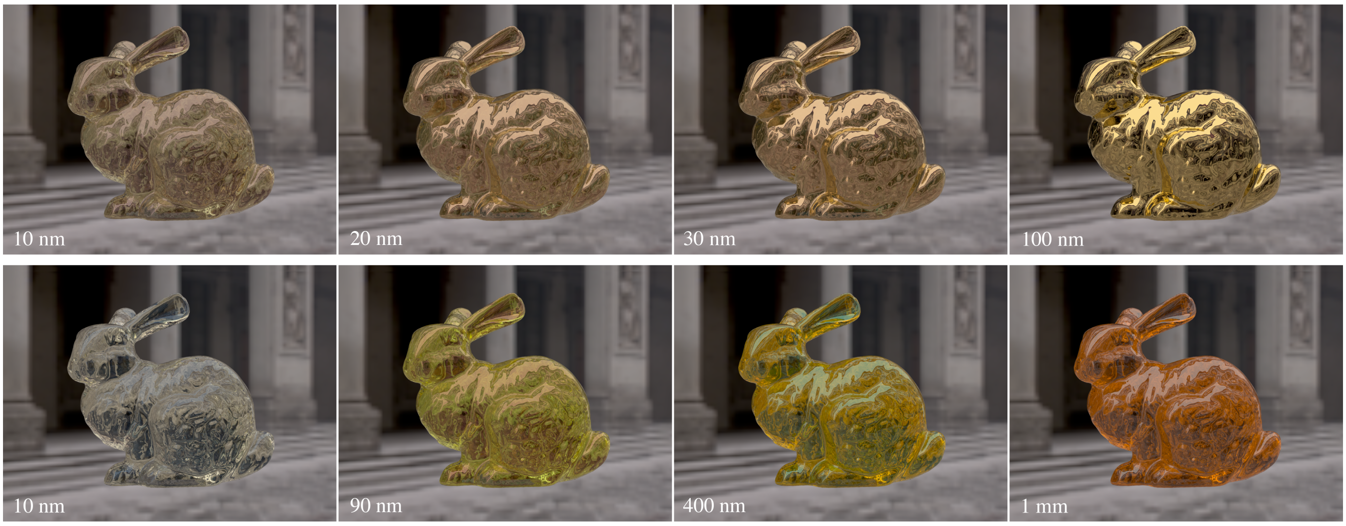


Figure 4: *Objects made with transparent glass and covered with a thin gold (top) or Cadmium sulfide (CdS, bottom) layer, with thickness increasing from left to right.*

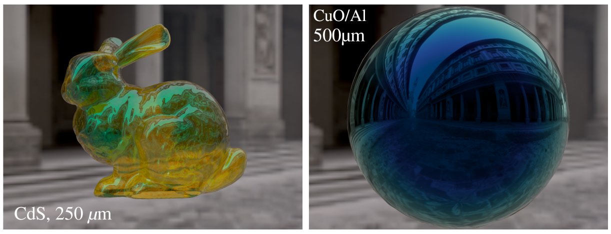


Figure 5: *Interference effects with thin layers. Left: air-Cadmium sulfide-air, thickness: 250 μm. Right: air-Copper Oxide (CuO)-Aluminium, thickness 500 μm.*

visible color changes depending on the surface orientation (see Figure 5). The effects are more visible with semi-conductors; with metals, the attenuation is too strong and light is absorbed before it has the chance to do several bounces inside the layer.

6. Conclusions

We have presented a model for rendering materials with a complex index of refraction. Examples include thin metallic layers but also many colored pigments. Our model reproduces the spectral variations caused by multiple bounces inside the layer: the color and aspect of the object change dramatically with the thickness.

Taking into account interference in pigment layers causes a dramatic change in hue, which corresponds to experimental measures. Our model is still too simple compared to reality: pigment layers are not made of a continuous layer; they are more likely to be an aggregation of small particles. This causes more color variations, due to the interference between the constituants. Similarly, the local structure of metallic layers also plays a role on their color appearance.

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