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Short Paper

External Forces Guided Fluid Surface and Volume Reconstruction from Monocular Video

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Abstract

We propose a novel method to reconstruct fluid's volume movement and surface details from just a monocular video for the first time. Although many monocular video-based reconstruction methods have been developed, the reconstructed results are merely one layer of geometry surface and lack physically correct volume particles' attribute and movement. To reconstruct 3D fluid volume, we define two kinds of particles, the target particles and the fluid particles. The target particles are extracted from the height field of water surface which is recovered by Shape from Shading (SFS) method. The fluid particles represent the discrete form of the 3D fluid volume and conform to the flow hydrodynamic properties. The target particles are used to guide the physical simulation of fluid particles based on the Smoothed Particle Hydrodynamics (SPH) model. To formulate this guidance, a new external force scheme is designed based on distance and relative motion between target particles and fluid particles. Additionally, in order to integrate and maintain geometric and physical features simultaneously, we adopt a two-scale decomposition strategy for the height field, and only apply the low frequency coarse-scale component to estimate the volumetric motion of liquid, while serve high frequency fine-scale component as noise to preserve fluid surface details in the stage of rendering. Our experimental results compare favorably to the state-of-the-art in terms of global fluid volume motion features and fluid surface details and demonstrate our approach can achieve desirable and pleasing effects.

CCS Concepts

• ***Computing methodologies*** → ***Physical simulation;*** *Volumetric models;*

1. Introduction

Modeling and reconstructing 3D fluid volume is a challenging task for two reasons. First, accurately measuring 3D fluid volume is difficult because its features can be highly varying in space and time, and the measuring device should not be intrusive to the fluid. Second, the evolution of fluid volume is governed by complex physical mechanisms that need to be incorporated into the reconstruction methods.

Existing fluid reconstruction methods can be broadly categorized as physically-based approaches [IOS*14] [MUM*06] [SS17] and video-based approaches [Adr91] [IKL*10]. For the monocular video, the state-of-the-art researches combined shape from shading method with shallow water model [LPS*12] [WWQZ17]. But Li et al. [LPS*12] made it clear that the shallow water equation only can model the gentle waves. For the complex water movement, they proposed a multi-layer shallow water model to capture the waves of different scales. Even more regrettable was that they only reconstructed surface of water and ignored the kinetic characteristic of interior water. Although Wang et al. [WWQZ17] spread the reconstructed surface information to the entire 3D volume and prepared SPH particles $Volume_{SPH}$ to physically simulate the fluid movement, they utilized the volumetric particles $Volume_{video}$ in the next video frame to geometrically correct $Volume_{SPH}$. However, the position, velocity and acceleration of the $Volume_{video}$ didn't conform to the laws of physics, because that $Volume_{video}$ was filled by artificial volumetric particles whose physical quantities were obtained by interpolation. There is no significance to use the invalid $Volume_{video}$ to correct SPH particles $Volume_{SPH}$. As same as Li et al. [LPS*12], Wang et al. [WWQZ17] only combined SFS with shallow water model rather than directly coupled SFS with SPH. They only applied SPH to simulate the interaction between fluid and solid.

Our goal is to reconstruct the 3D fluid volume with surface details from a monocular video. We use fluid particles to discretize 3D fluid volume and introduce the concept of target particles which can be generated from the height field of fluid surface. This height field is estimated by the SFS from a single viewpoint video. Although the highly reflective and refractive properties of water surface normally violate the Lambertian condition of SFS, human perception of geometric inaccuracies on these surfaces are less obvious and we're more concerned the dynamics of the surface, such as how the

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waves evolve with time [Mur92] [LPS*12]. Moreover, The water scenes of our choice appear visually opaque because of their depth and the suspension of dirt, mud and air [PFH10]. Hence, SFS can capture a similar geometry appearance dynamic behavior of the water [PLC*10]. At present, traditional physically-based approaches for fluid animation drive the fluid movement with forces [TKPR06]. This gives us tremendous inspiration, but few works calculate the force on the fluid according to the movement of the fluid video. This is the enormous challenge to us. From another perspective, reconstructing 3D fluid volume can be considered as animating a fluid. Therefore, in this paper, we propose an external force scheme to guide the fluid volume reconstruction, and the target particles can act as local magnets to attract nearby fluid particles, and the movement of the target particles can be consider as be similar to wind forces to transport fluid particles along their moving path [MM13]. So we design two kinds of forces: the position traction force and the velocity guidance force, which can be integrated into the external forces computation during the SPH simulation. High frequency surface details are difficult to be reproduced by SPH simulation, and will affect the reconstruction accuracy instead. Therefore, we decompose the height field into low- and high-frequency parts to better reproduce surface fluids motion and preserve the surface details respectively. And only low frequency coarse-scale component serves as source of external forces. High frequency fine-scale component is fused in the rendering stage to enhance the surface details.

Our technical contribution is to reconstruct the visual plausible water volume sequences from a monocular video using a relatively accurate physics model. Compared to the existing approaches, our method has the following advantages:

1. We are the first to solve the problem of three-dimensional fluid reconstruction from monocular video by physically-based model. We innovatively construct the mapping between shape from shading method and SPH model. Thus the reconstruction of fluid volume is simply and perfectly incorporated into SPH model and is governed by precise physical mechanisms.

2. The external force scheme extends the fluid volumetric shape controllability of SPH model. By adding these external forces, SPH model can simulate a what-you-see-is-what-you-get 3D water volume from monocular video. Furthermore, the implementation is very simple and avoids the complex numerical optimization.

3. Adding details with random noise is a traditional approach, but it is a novel idea to regard high-frequency information based on height field as noise. By filtering the height field, the low frequency component can better control the motion of the fluid, and the high frequency component can be superimposed directly on the fluid rendered to recover the video details better.

2. Overview

Our system inputs a monocular video and outputs a realistic 3D fluid volume model. As shown in Figure 1, it contains three principal steps, the first uses SFS to acquire the height field of the fluid surface, the second designs external forces using low-frequency height field as target guidance and SPH as the underlying physics model, the third transforms high-frequency fine-scale component of height field to noise model to preserve fluid surface details.

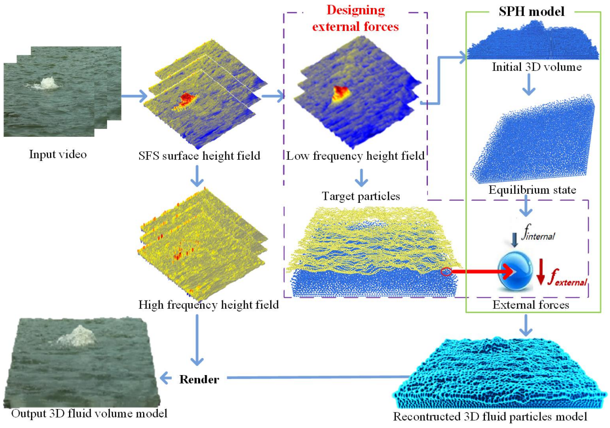

Figure 1: *Our system inputs a monocular video and outputs a realistic 3D fluid volume model. It contains three main steps: the first uses SFS to acquire the height field of water surface, the second designs external forces using SFS as target guidance and SPH as the underlying physics constraints, the third uses high-frequency height field to preserve surface details in the rendering stage.*

3. SPH fluid model based on external force scheme

In this paper, our main contribution is how to transform fluid 3D reconstruction problem to physically-based simulation problem. We define a fluid model which will be reconstructed to satisfy the SPH model property. This model includes three components, particle representation, fluid forces model, and surface details.

3.1. Particle representation

We define two kinds of particles, fluid particles and target particles. The 3D fluid to be reconstructed is represented by a set of interacting particles, which are called fluid particles. Fluid particles possess individual material properties and move according to the governing conservation equations. The target particles are only used to guide the movement of the fluid particles follow the laws of SPH simulation.

The target particles are extracted from the water surface which is reconstructed by SFS method and employed to guide fluid particles to achieve the desired the motion and appearance in the video. SFS obtains the 3D shape information by calculating the normal of object surface from illumination variation. In this paper, we follow [LPS*12] [WWQZ17] [PSS94] [PSS94] to recover the height field z. The important parameters of SFS are the tilt $\tau = 45^{\circ}$ and the slant $\sigma = 0.81^{\circ}$ of the illumination. Each frame in video can be discretized into particle form by pixel (x, y). The position of each particle can be represented as $R(x, y, z)$.

We first perform Gaussian filtering on the reconstructed height field to decompose the particles into low frequency field $R^{LF}(x, y, z)$ and high frequency field $R^{HF}(x, y, z)$. The value of filtering kernel is 9×9. After filtering the height field, the low- (the middle of bottom row) and high-frequency height field (the right of bottom row) are obtained. The low frequency height field greatly ameliorates the smoothness of target particles. In fact, low frequency height field is sufficient to characterize the general trend of the video fluid movement to make the guidance of the SPH simulation more stable. Therefore we only use the low frequency component to generate the target particles.

3.2. Fluid forces model

We choose Smoothed particle hydrodynamics (SPH) as underlying physics model. SPH model is a purely Lagrangian method that can be used to numerically solve the Navier-Stokes (NS) equations and can accurately describe the mechanical process. In SPH model, fluid is represented by a set of particles and the fluid dynamics are formed by the movements of all particles. For each particle i, its movement is driven by two kinds of forces, internal forces and external forces as Eq.(1).

$$f(i) = f_{internal}(i) + f_{external}(i) \tag{1}$$

SPH gives the formulas for calculating the internal forces [Mon92] [RL96], for instance, pressure force $f_{pressure}(i)$ and viscosity force $f_{viscosity}(i)$. But SPH doesn't have systematic formulas for the external forces on fluid particles. Here, we try to define the formulas of two external forces to drive fluid particles move according to the motion of target particles.

A target particle g is generated from the low frequency information $R^{LF}(x,y,z)$ of height field on fluid surface, and R_g denotes its position. Its velocity V_g is derived by the difference in the height field between two consecutive frames. However, this velocity only has non-zero value in the vertical direction. Because target particles are filled with entire $x-y$ plane of fluid particles, so only vertical velocities of target particles are sufficient to drive the movement of fluid particles.

Target particles serve as guidance to fluid particles in motion and appearance. Therefore, we assume that fluid particles can be pulled towards to target particles. But how to predict the motion of fluid particles is a challenging problem. We introduce Thurey et al.'s idea [TKPR06], which directly induces forces to attract the fluid or influence the velocity field. These forces are the external forces of fluid particle i, which are computed using target particles within the support domain of i.

Based on the distance between fluid particle and target particle, we design a position traction force $f_{position}(i)$ which is exerted on fluid particle i to follow the motion of target particle g.

$$f_{position}(i) = w_a \sum_g \alpha_g \frac{R_g - r_i}{||R_g - r_i||} W(d_{g,i}, h) \tag{2}$$

where w_a is a global constant that defines the strength of the traction force, $d_{g,i} = ||R_g - r_i||$ is distance between R_g and the position r_i of fluid particle i, h is smooth radius and W is target particle kernel function. We use a normalized spline kernel to be target particle kernel function as the same as the density approximation. And α_g is a scale factor which is defined for the position traction force as

$$\alpha_g = 1 - min(1, \sum_{i=1} \frac{m_i}{\rho_i} W(d_{g,i}, h)) \tag{3}$$

α_g scales down the position traction force when the influence region of target particle is already sufficiently covered with fluid. This preserves as much of the natural fluid behavior as possible.

The motion of target particle also has an effect on fluid particle. We define velocity guidance force $f_{velocity}$, which is dependent on velocity differences between target and fluid particles, to control this influence. It is similar to the viscosity force, but viscosity force is internal force between fluid particles. The velocity guidance force $f_{velocity}$ for the fluid particle i can be expressed as:

$$f_{velocity}(i) = w_m \sum_g (V_g - v_i) W(d_{g,i}, h) \tag{4}$$

where v_i is the velocity of fluid particle i, and w_m is a constant that defines a weight of the influence force.

Finally, the total force function for fluid particle i in Eq.(1) can be modeled as follows:

$$f(i) = f_{position}(i) + f_{velocity}(i) + G(i) + f_{pressure}(i) + f_{viscosity}(i) \tag{5}$$

Here $G(i)$, $f_{pressure}(i)$, and $f_{viscosity}(i)$ are the gravity, pressure, viscosity forces respectively from the SPH simulation. In our external force scheme, the neighboring particles of i can be divided into two groups: target particle g and fluid particle j. The forces between neighboring fluid particle j and i are internal forces, such as pressure and viscosity. And the forces between neighboring target particle g and i are external forces, i.e. $f_{position}(i)$, $f_{velocity}(i)$. Our hybrid fluid method is an elegant fusion of SFS and SPH.

3.3. Fluid surface detail

Low frequency height field can help to reconstruct the surface geometry and motion well coupling with the SPH simulation, but the result still looks smooth. However, surface detail is an important visual element in real fluids. To meet the demands of high fidelity of visual details in the reconstruction, we develop an efficient noise addition method to enhance fluid details [LXNT14].

The fluid surface details are represented in the high frequency parts of the reconstructed height field. The higher the frequency, the finer the details are. We propose an idea that the high frequency information is a way of generating noise, which can be used to add surface details by perturbing fluid particles on a small scale. The high frequency noise is directly used in the stage of fluid rendering.

At the stage of rendering, the rendering data consists of the position r_i of particles i, $i \in \{0, ..., n\}$. We take advantage of screen space fluid rendering approach [vdLGS09] which starts from the fluid particle positions. The value of the high frequency noise is multiplied with an exponential fall-off based on the depth below the surface to compute the pixel value $I(x,y)$ as Eq.(6), so that particles contribute less as they submerge,

$$I(x,y) = noise(x,y) * e^{(-x^2 - y^2 - (p_z(x,y) - d(x,y))^2)} \tag{6}$$

where p is the view-space position of pixel (x,y) with corresponding high frequency position $R^{HF}(x,y,z)$, $noise(x,y)$ is produced by high frequency information. d is the depth sampled from the surface depth texture, and x and y vary between -1 and 1.

4. Experimental results

We perform experiments on various fluid video scenarios in the public Dyntex dataset [PFH10] captured by an ordinary camera. All simulations were run on a PC equipped with an Intel(R) Core(TM) i7-7700 3.6 GHz processor, NVIDIA GeForce GTX 1070 8BG display card, and 16GB RAM.

By introducing two new external forces between target particles and fluid particles, we skillfully couple SFS method with SPH model . In the pure SPH simulation, we set a large time step $\Delta t = 0.01s$ to increase efficiency. When the movements of fluid particles reach equilibrium, we add the position traction force and the velocity guidance force to SPH model. At this stage, the time step should take a smaller value such as $\Delta t = 0.001s$ to make sure the external forces work long enough. When updating target particles, the time consistency between video data and SPH simulation should be taken into account. We represent the time interval between two adjacent frames using T which is numerically equal to the inverse of the video frame rate (25fps). And its rate of change with time is symbolized by $\Delta T = \frac{1000}{25Hz} = 0.04ms$. We establish the formula for the update rate Ψ according to video frame rate and SPH time step Δt as $\Psi = \frac{\Delta T}{\Delta t} = 40$. It means that every Ψ iterations of fluid particles, video runs to the next frame and the position of target particle is updated from $R_g(T)$ to $R_g(T+1)$. Additionally, in Eq.(2, 4), we adopt $w_a = \frac{0.5}{h}$, $w_m = \frac{0.015}{h}$ to compute the external forces (h denotes the smooth radius). The same parameters are used for every example in our paper.

As shown in Figure 2, we compare the reconstructed 3D fluid volume (right column) with ground truth (left column) which from the SPH-based simulated result. They maintain high consistency in fluid motion and geometric appearance. We calculate the depth errors of two kinds of particles by:

$$e = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(z_i^{GT} - z_i^{RE})^2} \qquad (7)$$

where z_i^{GT} and z_i^{RE} are the perpendicular component of the i particle position of the ground truth and the reconstructed fluid, respectively. The results are listed in Table 1. It shows that the reconstruction results are close to the real values.

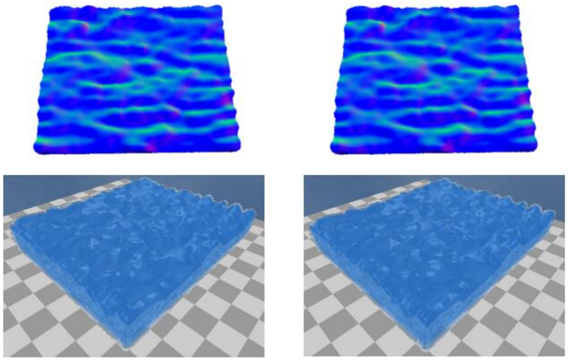

Figure 2: *Comparison of the reconstructed 3D fluid volume (right column) with ground truth (left column).*

Table 1: *Statistics result of the depth errors*

No.	error	No.	error	No.	error	No.	error
1	0.0379	6	0.0367	11	0.0376	16	0.0393
2	0.0389	7	0.0379	12	0.0381	17	0.0388
3	0.0379	8	0.0380	13	0.0373	18	0.0384
4	0.0376	9	0.0377	14	0.0381	19	0.0369
5	0.0389	10	0.0382	15	0.0382	20	0.0384

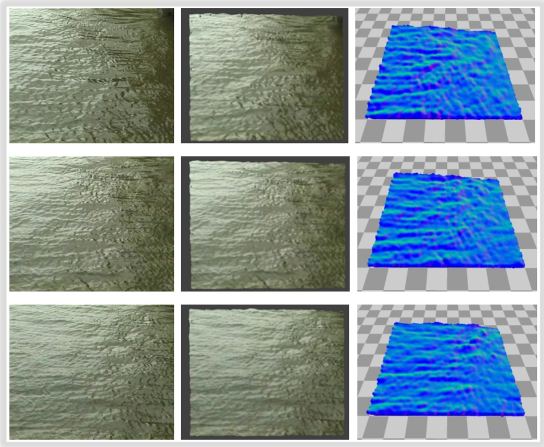

Figure 3: *The reconstructed movement and surface of calm water scene*

Our method has a good effect on both calm waves and complex fluid movements. As shown in Figure 3, we provide the result of Gentle Waves, along with its reference input video (the first column) for comparison. The results in second column are rendered with textures from the original video. The third column gives a opaque rendering. Furthermore, the second and third rows show the movements and surface details of reconstructed Gentle Waves from three different video frames. As the same, Figure 4 shows Fountain example with complex water movement. Although SFS cannot generate real breaking waves nor foams or splashes, the water surfaces still look plausible with the help of textures from the input video (the second column) and the movements of Fountain (the third column) are highly consistent with the video (the first column) through Gaussian filtering. We also provide the other reconstruted results in Figure 5.

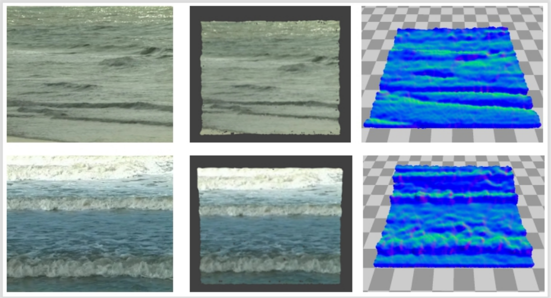

Figure 5: *The reconstructed movements and surfaces of Slight Waves (the top row) and Surge (the bottom row).*

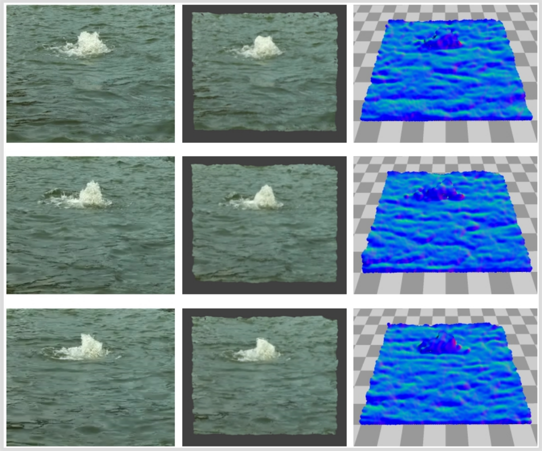

Figure 4: *The reconstructed movement and surface of complex water scene.*

In particular, as shown in Figure 6, we verify that our detail-preserving approach clearly increases the fine-scale fluid appearance by adding the high frequency information (right). Compared to the input video (left), the low frequency parts (middle) of the height field successfully guide fluid particles to follow the motion of target particles. However, the low frequency components merely reconstruct the overall shape of 3D fluid volume. The surface of water volume is excessive smooth. As shown in right sub-Figure, the high frequency fine-scale component can effectively express complex fluid details.

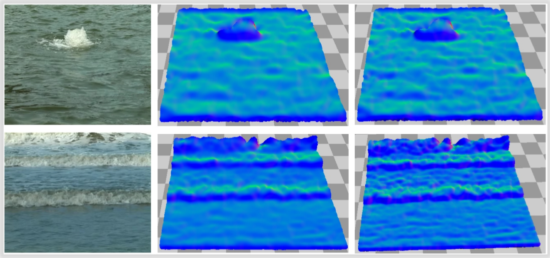

Figure 6: *Detail-preserving and visual comparisons between video frame (left) and reconstructed fluid. We show the results of reconstruction from low-frequency information (middle) and the detail-preserving by adding the high frequency parts (right).*

It is worth comparing with Li et al. [LPS*12], which as far as we know is the latest method to recover water surface using SFS and shallow water model. They used SFS to acquire an initial water surface and estimated a three-dimensional flow field using shallow water as the underlying physics constraints. They made it clear that the one-layer shallow water equation wasn't adequate to cope with complex water movement. Furthermore, the shallow water model is impossible to reconstruct three-dimensional water volume. To compensate for the inaccurate calculation of their underlying physics, we compute the dynamic fluid motion field by SPH method in which the estimated velocities comply with the conservation of mass in 3D instead of shallow water model. As shown in Figure 7, we extract the surface (the middle) from the reconstructed 3D fluid volume and compare with Li et al. [LPS*12] (the right). While the geometric appearance and movement of our reconstructed water surface is visually similar to that of Li et al. [LPS*12], we fruther reconstruct the entire 3D fluid volume. Our work is more challengeable and meaningful.

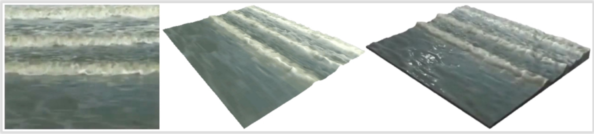

Figure 7: *Comparison of 3D fluid reconstruction. The left shows input video, the right shows Li et al.'s result, the middle shows our result.*

We compare our method with Wang et al. [WWQZ17] , which as far as we know is the latest method that simultaneously reconstructed water surface and volume from a single viewpoint video. In their SPH simulation, they utilized the volumetric particles $Volume_{video}$ in the next video frame to correct $Volume_{SPH}$. However, the position, velocity and acceleration of the $Volume_{video}$ didn't conform to the laws of physics, because $Volume_{video}$ was filled by artificial volumetric particles and their physical quantities were obtained by interpolation. There is no significance to use the invalid $Volume_{video}$ to correct SPH particles $Volume_{SPH}$. As same as Li et al. [LPS*12], Wang et al. [WWQZ17] only combined SFS with shallow water model rather than directly coupled SFS with SPH. They only applied SPH to simulate the interaction between fluid and solid. They combined three methods including SFS, shallow water model and SPH. Mixing the three methods adds complexity to fluid reconstruction. And the implementation process is redundant. We discard the shallow water model and directly reconstruct fluid coupling with SFS and SPH. Figure 8 shows the comparison between ours and [WWQZ17]. Our reconstruction results retain more accurate details and are more similar to the input video.

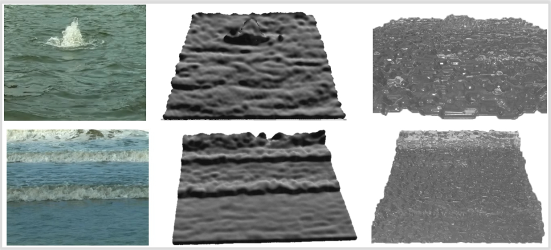

Figure 8: *Comparison of 3D fluid reconstruction. The left column shows input videos, the right column shows Wang et al.'s results, the middle column shows our water reconstruction.*

5. Conclusions

We have detailed a novel approach for 3D dynamic fluid reconstruction problem: simulating fluid volume under the guidance of water surfaces. For the given monocular video example, we present an external force scheme to reconstruct fluid's motion and a two-scale decomposition strategy to preserve surface details. This hybrid fluid method coupling SPH with SFS is an extremely simple algorithm which can be implemented by using the framework of SPH. To our best knowledge, this is the first approach that transforms the video-based fluid 3D reconstruction problem into the external force scheme of the physically-based method. We have illustrated various types of experiments and demonstrated that our proposed method is applicable for generating motion features and surface details of fluid 3D volume.

Limitation and Future work With the limitation of SFS algorithm, our method is not applied to fine water details (e.g., droplets, or splashes, or sprays), where don't exist meaningful height field. And because of the reflective and refractive properties of water, SFS can only recover an approximate appearance shape, can't accurately acquire a water surface. We plan to address this issue in future research, and try to build neural network to directly learn the attributes (i.e. height field, pressure, or velocity) of the video fluid. In addition, there is undesired noise on our reconstructed surface. We will attempt to solve it by increasing particle resolution or optimizing the surface extraction algorithm. Moreover, the depth of water volume is limited to the numbers of particles because correspondence searching of neighboring particles is expensive cost. No matter how deep the water is, we plan to achieve better performance with the help of GPU. Finally, WCSPH imposes a severe time step restriction and the computational expenses of simulating high resolution fluid animations are too large. Therefore, we plan to adopt algorithmic improvements e.g. PCISPH [BET*10], IISPH [ICS*13], ISPH [TDF*15], DFSPH [BK16], etc. to further reduce the cost of the fluid reconstruction system.

Acknowledgement

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