Minimising Longest Edge for Closed Surface Construction from Unorganised 3D Point Sets

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Abstract

Given an unorganised 3D point set with just coordinate data, we formulate the problem of closed surface construction as one requiring minimisation of longest edge in triangles, a criterion derivable from Gestalt laws for shape perception. Next we define the Minimum Boundary Complex *(BC_{min}), which resembles the desired surface B_{min} considerably, by slightly relaxing the topological constraint to make it at least two triangles per edge instead of exactly two required by B_{min}. A close approximation of BC_{min} can be computed fast using a greedy algorithm. This provides a very good starting shape which can be transformed by a few steps into the desired shape, close to B_{min}. Our method runs in $O(n \log n)$ time, with Delaunay Graph construction as largest run-time factor. We show considerable improvement over previous methods, especially for sparse, non-uniform point spacing.*

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Boundary representations I.4.8 [Computer Graphics]: Scene Analysis—Surface Fitting

1. Introduction

Defining the piece-wise linear boundary shape for a solid object in $\mathbb{R}^3$, for which only the surface point coordinates but not their topology is known, is difficult. Such unorganised point sets are increasingly encountered as output of 3D scanners, etc., since surface normals cannot be reliably estimated at these points. Further, these point sets are also not uniformly dense everywhere. Yet, generating water-tight surfaces for such point sets is important in practice.

Previous methods for reconstructing a surface exclusively from point coordinates often rely on a sampling criterion, which was first defined by Amenta et al. [ACDL00]. Such criteria give a guarantee that a point set is interpolated locally by a unique surface (homeomorphy). However, their nature requires restricting of the maximal surface curvature severely to ensure such a unique fit. In practice, sampling-based algorithms are applied to point sets outside these theoretical limits, but fail often for non-uniformly spaced points, even with additional hole-filling operations.

Ohrhallinger and Mudur [OM11] formulate the reconstruction of a shape in $\mathbb{R}^2$ as minimisation of a criterion which is related to the Gestalt laws of *Proximity*, *Good Continuity* and *Closure*, and we extend it here into $\mathbb{R}^3$.

2. Algorithm

Minimising curvature relates well to the Gestalt laws, but it depends on all triangles incident at a point on the interpolated surface. To avoid combinatorial dependency, we prefer a criterion which can be independently computed per triangle. Our experiments have shown that using the criterion of minimising $\lambda(t)$ as the *longest edge* in a triangle t yields boundary shapes, more aesthetically pleasing when compared to other triangle measures, such as area or circumradius. We believe that this is due to the fact that short edges also tend to minimise *mean curvature* $H_e = 2\|e\| cos \frac{\theta_e}{2}$. Clearly, determining this minimal interpolating surface is also NP-hard and appropriate heuristic techniques are needed. We start with the *minimum boundary complex*:

Definition 1 $BC \subset DG$ in $\mathbb{R}^3$, BC is a set of triangles spanning P and each vertex $v_i \in BC$ has ≥ 1 umbrellas in BC.

$$BC_{min} = \sum_{t_i}^{T} \lambda(t_i) \rightarrow \min$$

The *hull* $H(T)$ for a connected set of triangles $T \in DG$ is defined as its sub-set such that traversing $H(T)$ in consistent orientation contains all triangles in $T \setminus H(T)$ at one side, called the inside. $H(T)$ is manifold if at every vertex in $H(T)$ the incident triangle fan forms exactly 1 umbrella.

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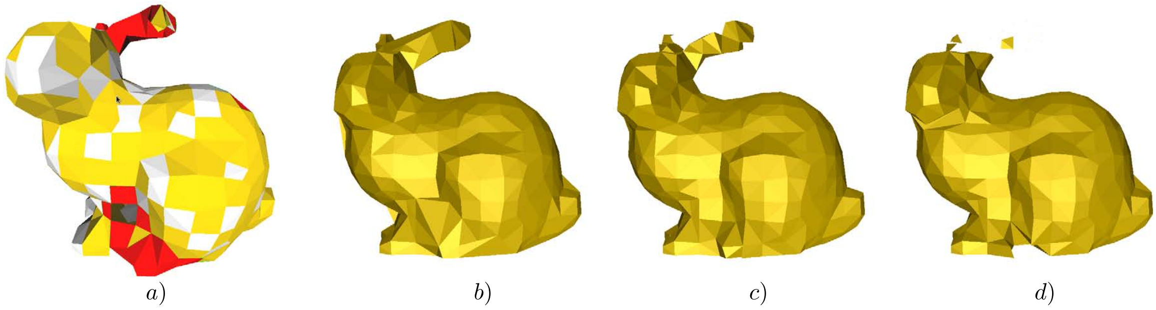

Figure 1: *Our algorithm in a nutshell: a) Minimum Boundary Complex, tetrahedra as white, or red if bounding a hole. b) Inflated to manifold hull. c) Sculptured to interpolating boundary. d) Previous best method of Dey and Goswami [DG03].*

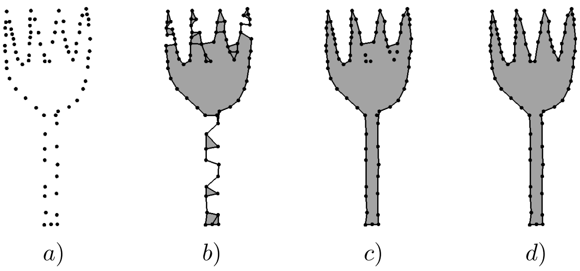

Figure 2: *Equivalent in $\mathbb{R}^2$: The inside of the hull H is always shaded grey: a) Point set. b) BC. c)* Inflated *to BC' with manifold hull. d)* Sculptured *to interpolating boundary B_{min}.*

A close approximation of BC_{min} can be constructed using a simple greedy algorithm in $O(n \log n)$ time. This *simplicial complex* is a good starting shape because it resembles B_{min}, having many triangles in common and of the same genus.

Sculpturing as introduced by [Boi84] removes tetrahedra from a simplicial complex $\mathbb{C}$, sorted by an intrinsic criterion. to expose interior vertices onto the boundary $B(\mathbb{C})$. Sculpturing from the convex hull of a point set runs quickly into local minima. Instead we propose to start from the boundary complex BC, which as we said earlier closely resembles the desired shape. But BC is not guaranteed to be manifold. So, we introduce a step before sculpturing called *inflating*, the *dual* to sculpturing. It adds tetrahedra to the BC, sorted by an intrinsic criterion, until the resulting BC' is such that its $H(BC')$ is *manifold*, possibly making some points *interior*.

Table 1 shows the tightening of topological constraints for the two steps, as is illustrated in Figure 2 for $\mathbb{R}^2$:

Step	Simplicial Complex	Constraint
Construction	BC	$v \geq 1$
Inflating	$H(BC')$	$v = 0 \vee v = 1$
Sculpturing	B_{min}	$v = 1$

Table 1: *A comparison of the described simplicial complexes by their constraints, their per-vertex umbrella count.*

Even the inflating operation can terminate, as does sculpturing, at local minima if the starting simplicial complex is very different from B_{min}. This is specifically so in regions where large tetrahedra should be added before smaller ones, e.g. a sphere with a small hole, so we apply a *hole-filling* operation prior to the inflating step. Details of the hole-filling, inflation and sculpturing operations (see Figure 1) are omitted here but will be provided in a full paper version under preparation. The point set in this example is a small one, but with bad spacing, which makes correct surface construction difficult even for the previous best working algorithm. Lack of space prevents us from showing larger point sets.

3. Conclusion

We define Minimum Boundary Complex, a shape which closely resembles the desired closed surface interpolating an unorganised 3D point set. A greedy algorithm computes an approximation fast. It is then transformed into the desired shape by three steps, hole-filling, inflation and sculpturing. We get much improved quality of shape construction and competitive computational complexity when compared with the best known previous method [DG03]. At the core, our method avoids local minima effectively through stepwise tightening of a topological constraint.

References

[ACDL00] Amenta N., Choi S., Dey T. K., Leekha N.: A simple algorithm for homeomorphic surface reconstruction. In *SCG '00: Proceedings of the 16th ann. symp. on Comp. geometry* (New York, NY, USA, 2000), ACM, pp. 213–222. 1

[Boi84] Boissonnat J.-D.: Geometric structures for three-dimensional shape representation. *ACM Trans. Graph. 3*, 4 (1984), 266–286. 2

[DG03] Dey T. K., Goswami S.: Tight cocone: a water-tight surface reconstructor. In *SM '03: Proc. of the 8th ACM symp. on Solid mod. and app.* (New York, 2003), ACM, pp. 127–134. 2

[OM11] Ohrhallinger S., Mudur S. P.: Interpolating an unorganized 2d point cloud with a single closed shape. *Computer-Aided Design 43*, 12 (2011), 1629–1638. 1