Pedagogic discrete visualization of electromagnetic waves

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Abstract

A dynamic electromagnetic wave propagation visualization tool is presented in this paper. It can simulate wave reflection and diffraction in 2D. It allows to visualize the electric field magnitude. The original approach is based on discrete analytical geometry allowing a dynamic visualization of the wave propagation contrary to existing models. We use it for the moment as pedagogic tool for the understanding of this complex phenomenon.

Categories and Subject Descriptors (according to ACM CCS):
I.3.3 [Computer Graphics]: Applications

1. Introduction

This paper deals with electromagnetic wave propagation simulation and visualization. The finite difference method in time domain is the reference model[2] for computing the wave propagation at a given time. It proposed however only a static visualization: for every point of the scene (defined as a grid), the electromagnetic energy is computed for each given transmitter position at a given time. The outcome is the electromagnetic energy loss in the scene, visualized by a color gradation. This representation is not suited for a pedagogic visualization of complex phenomena such as reflection, diffraction and interference. Students have often difficulty to understand the dynamics of such wave propagation phenomena. Our purpose in this paper is not to propose a more precise computation method. It is to offer a new visualization tool that allows to dynamically observe wave propagation phenomena.

In this article, we suggest a new and explicit way to represent electromagnetic wave propagation. This dynamic description constitutes an interesting pedagogic tool for the understanding of wave propagation mechanisms. What is specific in our work, is the using of discrete analytical geometry to realize the discretization of the wave propagation mechanisms. Whatever it deals with -lines, planes, circles, spheres- discrete analytical geometry allows a control over the properties of the discretized primitives. Moreover, they can be drawn by robust and efficient incremental algorithms.

In the following part, we mention quickly a few notions about electromagnetic wave propagation, and we give the basic definitions of the the discrete and analytical geometry that we use. Then we explain how we discretize a wave propagation, before presenting our recent results. Finally, we discuss about what could be at stake with such a method, and we give our conclusions.

2. Preliminaries

Physics view an electromagnetic wave as a vectorial value, which is periodic in the space and time domain. This wave theory[1] fully characterize a propagation with Maxwell's equations. Any study of a propagation problem comes down to solve Maxwell's equations in a given point and for a given instant. The reference model to simulate wave propagation by computer is based on Maxwell's equations. It is called "finite differences in time domain" [2]. This technique discretizes the scene in regular cells and for each of them, numerically solves Maxwell's equations. We get for each cell the electric and magnetic field values, represented by a gradation of colors, as mentioned in introduction. The wave theory explain interferences between waves: when at least two waves are superposed in a point, the result is the sum of those two signals. The outcome are a loss (due to destructive interferences) or an amplification (due to constructive interferences) of the electric field magnitude.

These methods are physically very precise but they do not allow an explicit visualization of all mechanisms connected to wave propagation. It is not very easy for a student to dif-

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ferentiate multiple reflections and diffractions on such images. Our purpose is to propose a visualization tool to illustrate such phenomena explicitly for a pedagogic use.

We will have to use geometrical optic and its extensions, that approximize Maxwell's equations in high frequencies by assimilating a wave to a ray. Descartes' laws can then be applied and characterize the reflection of a wave against a plane surface, or its refraction. However if geometrical optic can highlight a phenomenon such as diffraction, it can not explain it. This phenomenon is described by the uniform diffraction theory [3]. It appears when a wave meets with an object whose dimensions can be compared to its wavelength. This one behaves as a new transmitter, producing a secondary wave with the same characteristics.

In this article, we consider a 2D outdoor propagation in an homogeneous environment. According to these hypothesis, the speed of the propagation is the same whatever the direction. The wave theory states that the wave shape is circular. We will also use geometrical optic and its extensions in order to define geometrically a discrete wave front, reflected or diffracted as we do not treat refraction. For each point (of a grid) of each wave, the value of its electric field is computed in order to visualize interferences.

2.1. Discrete Analytical 2D straight lines and circles

The 2D scene is defined as a set of convex polygons. Their discretization is based on the analytical discrete hyperplane [4 5] defined on $\mathbb{Z}^n$ by :

$$A \leq \sum_{i=1}^{n} \alpha_i X_i < B$$

with $\alpha_i, A, B \in \mathbb{R}$. $w = B - A$ is the hyperplane thickness. In particular a n dimensional hyperplane is naive if its thickness verifies $w = max(|\alpha_i|)$. We discretize the polygons edges with discrete analytical lines. The Euclidean line $d : ax + by + c = 0$ corresponding to an edge of the polygon P, is discretized by the set of $\mathbb{Z}^2$:

$$-\frac{max(|a|,|b|)}{2} \leq ax + by + c < \frac{max(|a|,|b|)}{2}$$

Considering the two half-planes $ax + by + c \leq \frac{max(|a|,|b|)}{2}$ and $ax + by + c \geq -\frac{max(|a|,|b|)}{2}$, we call d_+ the straight line of equation $ax + by + c = \pm\frac{max(|a|,|b|)}{2}$ that delimits the half-plane that contains P.

To discretize wave fronts, we use discrete analytical hyperspheres [6], which are defined in n dimensions by:

$$A \leq \sum_{i=1}^{n} (X_i - c_i)^2 < B \text{ with A, B} \in \mathbb{R} \text{ and c} \in \mathbb{R}^n$$

where $w = B - A$ is the thickness of the hypersphere. In our case, we only use circles with integer radius and thickness equal to one, which corresponds to connex circles with minimal thickness. A wave front with center $(c_x, c_y) \in \mathbb{R}^2$ and with radius $R \in \mathbb{N}$ is thus defined analytically by the set of points $(x, y) \in \mathbb{N}^2$ verifying :

$$R^2 \leq (x - c_x)^2 + (y - c_y)^2 < (R+1)^2$$

These circles can be drawn efficiently by an incremental algorithm. This definition presents two essential properties. The first one is that it accepts arbitrary Euclidean coordinate centers. The second one is that the layout of concentric circles whose radius increase one by one, forms a paving of the 2D plane. This is not the case with Bresenham circles. We are thus sure not to miss any points during a wave propagation simulation.

3. Discrete wave propagation

In this section, we describe the discrete objects and mechanisms leading to a new type of visualization for electromagnetic wave propagation. We will speak of real or discrete wave front without discrimination.

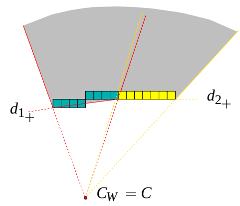

Figure 1: *Unions of discrete shadows.*

Let's first start without taking into account any reflection or diffraction. We consider a wave transmitter located in $C = (c_x, c_y) \in \mathbb{R}^2$. A discrete wave front W is a 2D analytical hypersphere with unit thickness. Its center C_W is merged with C. The propagation of a discrete wave is realized by drawing successive discrete wave fronts. Only $\mathbb{Z}^2$ points visible from C_W must be reached by the wave fronts. In a continuous 2D space, the shade area induced by an edge E is basically described by three straight lines: The line d of the edge, and the lines defined by C_W and the two vertices of E. This is transposed in the discrete domain as follows. We consider the line d_+ and the two Euclidean straight lines passing through one of the four vertices of each vertex pixel of the discrete edge, chosen in order to maximize the generated shadow. This insures the fusion of shadows for two incident edges, as it can be seen on the figure 1.

The visibility area of W is the complementary to $\mathbb{Z}^2$ for the union of the discrete shadows generated by the edges in the scene, in regard of C_W. Only the edges whose orientations point out the half-plane including C_W need to be taken into account. These edges will generate reflective waves.

3.1. Taking into account reflection

When a discrete wave W meets an edge E_r of a polygon, a new reflected wave W_r is created. Its virtual center C_{W_r} is the

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symetric of C_W in relation to the straight line d corresponding to E_r, in accordance with Descartes' laws. Let us emphasize that to use this definition, we need a discrete circle definition that allows arbitrary continuous centers. W_r is set up into the scene when the radius of W equals the integer part of the minimal distance between C_W and the edge pixels. The initial radius of W_r equals this distance. Now we have to find out how the visibility area of W_r can be computed. For this, we will define the initial shadow of a reflected wave. Firstly, let us pretend that E_r is completely visible. Then the initial shadow of W_r is defined by d_+, and the two lines crossing C_{W_r} and the edges' vertices (figure 2).

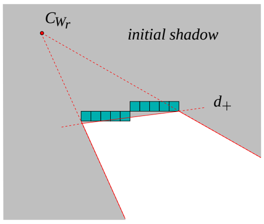
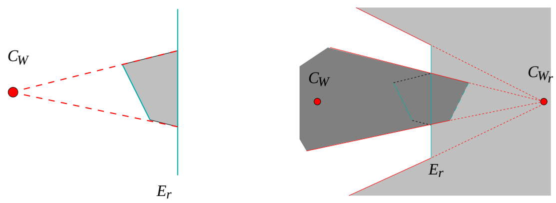


Figure 2: *Initial shadow of a reflected wave.*

Now, let us suppose that E_r is not completely visible (figure 3(a)). We have to add to the initial shadow, the symmetric in relation to d, of the shadows hiding E_r, as shown on the picture 3(b). In these two cases, the visibility area of W_r is computed with the complementary of the initial shadow in $\mathbb{Z}^2$.



(a) (b)

Figure 3: *Initial shadow of a reflection with a partially visible edge*

3.2. Taking into account diffraction

The interaction of a discrete wave W with the vertex S of a polygon P generates a diffracted wave W_d. Its center C_{W_d} is merged with S, its initial radius is null. The diffracted wave is set up when W strikes S, *e.g* when its radius equals the integer part of the distance from C_W to S. W_d does not interact with P again.

Let p be the pixel containing S and the two corresponding incident discrete edges obtained by discretization of d_1

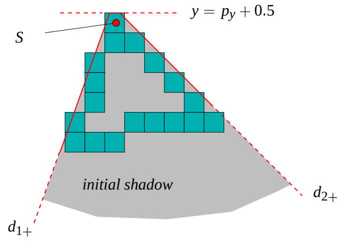


Figure 4: *Initial shadow of a diffracted wave.*

and d_2. We define the initial shadow of W_d by the area delimited by the straight lines: d_{1+}, d_{2+}, and depending on the orientation, the straight line $y = p_y \pm 0.5$ or $x = p_x \pm 0.5$. Figure 4 shows a configuration example. The initial shadow completely contains the polygon on which W_d has diffracted, and prevents any interaction with this one. Then, the visibility area of W_d is computed in the complementary to $\mathbb{Z}^2$ of this initial shadow.

4. Results

The implementation of the previous mechanisms allows a discrete dynamic and explicit visualization of the wave propagation. There are two types of representations: The geometric visualization with only the shape of the wave fronts, and the energetic visualization where color gradations show the magnitude variations of the electric field for each point reached by a wave. We use the extensions of the geometrical optic to compute the electric field value and to take into account the surface properties.

Let us now present and comment some pictures from animations obtained with our tool.

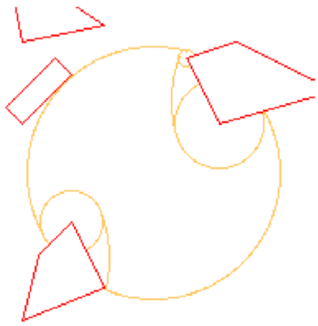


Figure 5: *Wave front visualization*

Figure 5 illustrates a discrete propagation where only geometric wave fronts are shown. We can see that each circle is drawn in its visibility area as expected. This animation presents wave fronts in motion and explicitly describes the reflection and the diffraction. This illustrates in a very intuitive way these mechanisms. This constitutes an interesting pedagogic visualization.

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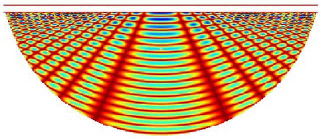

Figure 6: *Electric field visualization of a reflected wave on a not perfectly reflexive surface*

Each wave is independent, so it is possible to visualize only a chosen phenomenon. It is the case for the reflection with the figure 6. This animation shows the magnitude variations of the electric field for a reflection. We can observe the magnitude variations due to the surface properties and the privileged directions for the reflection of electric energy.

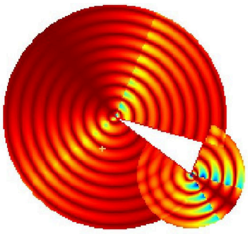

Figure 7: *Electric field visualization of a diffracted wave*

Figure 7 isolates the diffraction phenomenon. We verify that the initial shadows of the two diffracted waves prevent them to create new diffractions on the polygon vertices. The alternation of dark and light circular areas is the expression of the periodic nature of the electric fields. As for the reflection, color variations for each circle mean that the energy repartition is non-uniform. This example illustrates how points invisible from the transmitter can be reached by a radio wave.

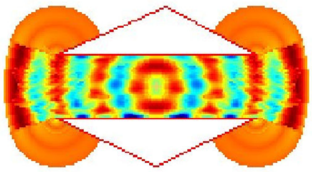

Figure 8: *Interferences between wave fronts*

At last, the figure 8 takes into account all mechanisms connected to the wave propagation. It mainly shows the interferences due to the different wave fronts. Moreover, this example and the two previous ones express the paving of the discrete plane by analytical circles. Each picture leads to the conclusion that our analytical definitions are reliable and allow a controlled discretization of the scene and of the visibility areas for each waves.

5. Conclusions and perspectives

This work is based on an original idea: to mix electromagnetic wave propagation and discrete analytical geometry. Interactions such as reflections and diffractions are not trivial phenomena and our tool is able to take them into account. Our first results show that discrete analytical geometry can illustrate physical phenomena in a new way that is particularly well suited for pedagogic applications. The fact that discrete analytical geometry allows a precise control on the discretization of Euclidean objects is a major advantage of the proposed method. Moreover, it preserves physical properties when going from the continuous to the discrete world. For the moment, we already have a pedagogic tool that has been successfully used in classrooms of physic in our electronic engineering department.

Now we are considering an extension to 3D wave propagation simulation. All the proposed definitions can be easily extended to 3D. However, diffraction on 3D edges do not exist in 2D. At the authors knowledge, physics do not foresee exactly what the shape of the wave front is. It could be a non-spherical shape. We'll have the problem of generating, defining and keeping a space paving property for such a discrete shape.

Finally, putting the pedagogic part aside, another step will be done to get an estimation of the physical precision of our method in regard of finite differences in time domain. This could lead to a broader area of applications.

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