Tutorial

Symmetry in Shapes Theory and Practice

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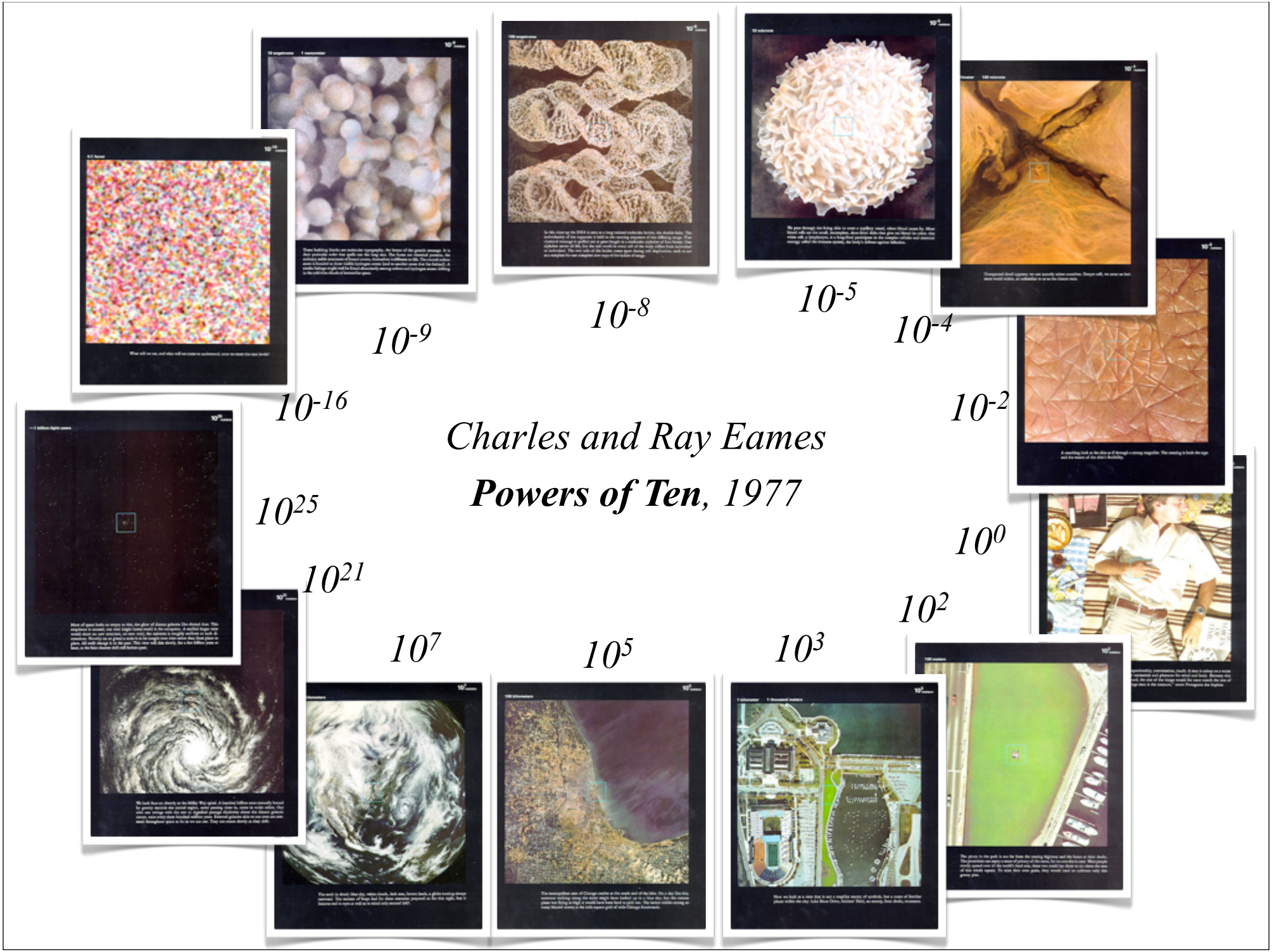
Geometry

γεωμετρία

geo = earth

metria = measure

"The branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space."


10^{-16}
10^{-9}
10^{-8}
10^{-5}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{3}
10^{5}
10^{7}
10^{21}
10^{25}
Charles and Ray Eames
Powers of Ten, 1977

Symmetry

συμμετρία

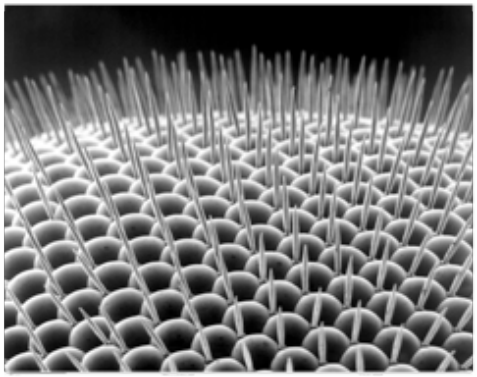
1. "similarity, correspondence, or balance among systems or parts of a system

2. "an exact correspondence in position or form about a given point, line, or plane"

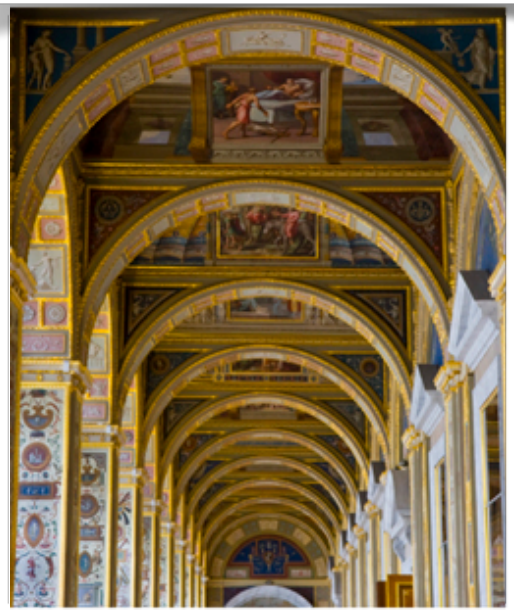
3. "beauty or harmony of form based on a proportionate arrangement of parts"

Collins English Dictionary

Symmetry



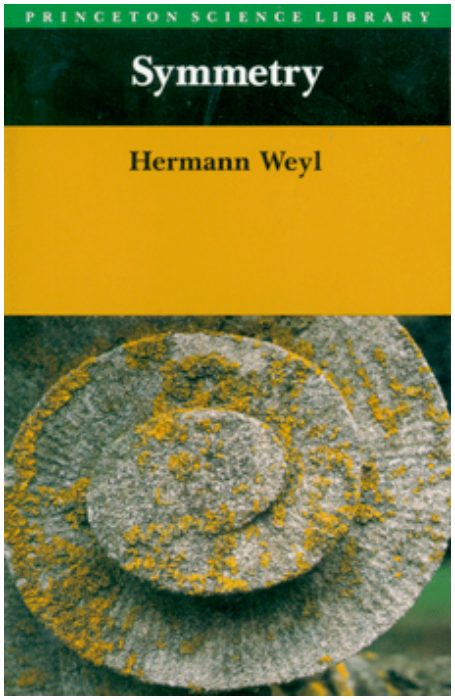
Symmetry



Symmetry

Group Theory

- Mathematical language of symmetry



H. Weyl, *Symmetry*. Princeton University Press, 1952

Transformations

Translation

Scale

Rotation

Symmetry Groups

Symmetry as ***invariance to transformations***

Rotation by $\frac{360°}{5} = 72°$

$2 \cdot \frac{360°}{5} = 144°$

$3 \cdot \frac{360°}{5} = 216°$

$4 \cdot \frac{360°}{5} = 288°$

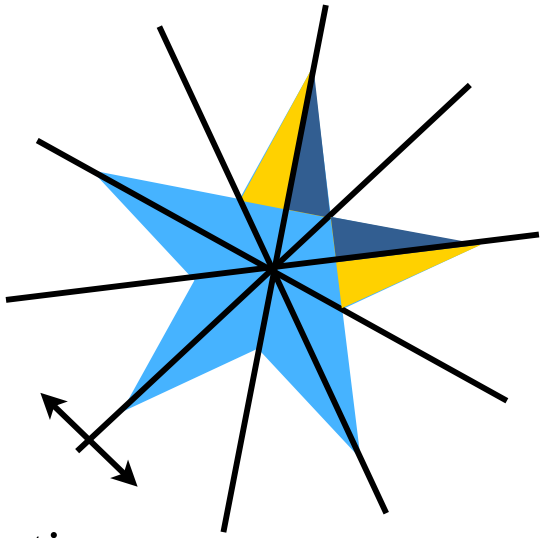
$5 \cdot \frac{360°}{5} = 360° = 0°$

↓

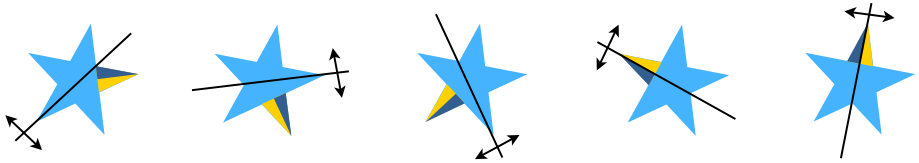
Cyclic Group C_5

Symmetry Groups

Symmetry as ***invariance to transformations***



Reflection



Rotation by $\frac{360^\circ}{5} = 72^\circ$

$2 \cdot \frac{360^\circ}{5} = 144^\circ$

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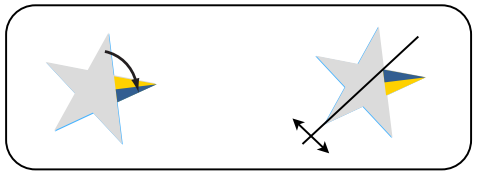
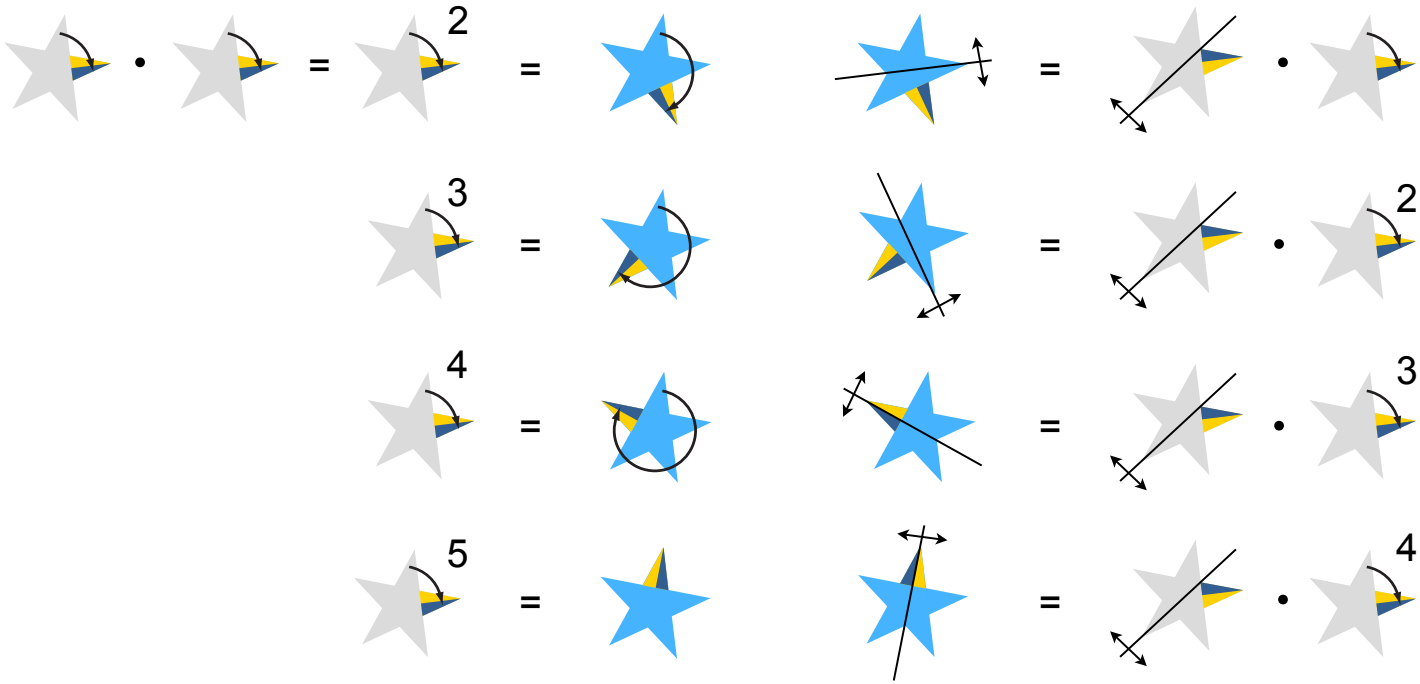
Dihedral Group D_5 ← **Cyclic Group** C_5

Symmetry Groups

Group Generators **Dihedral Group D_5**



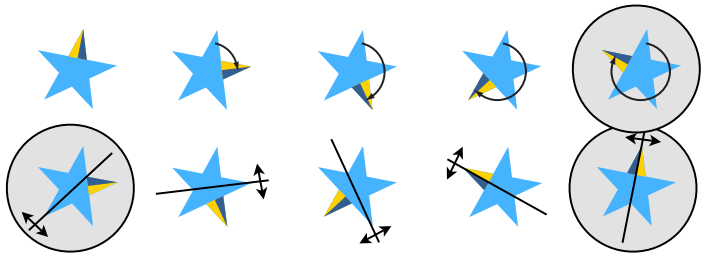
generating transformations



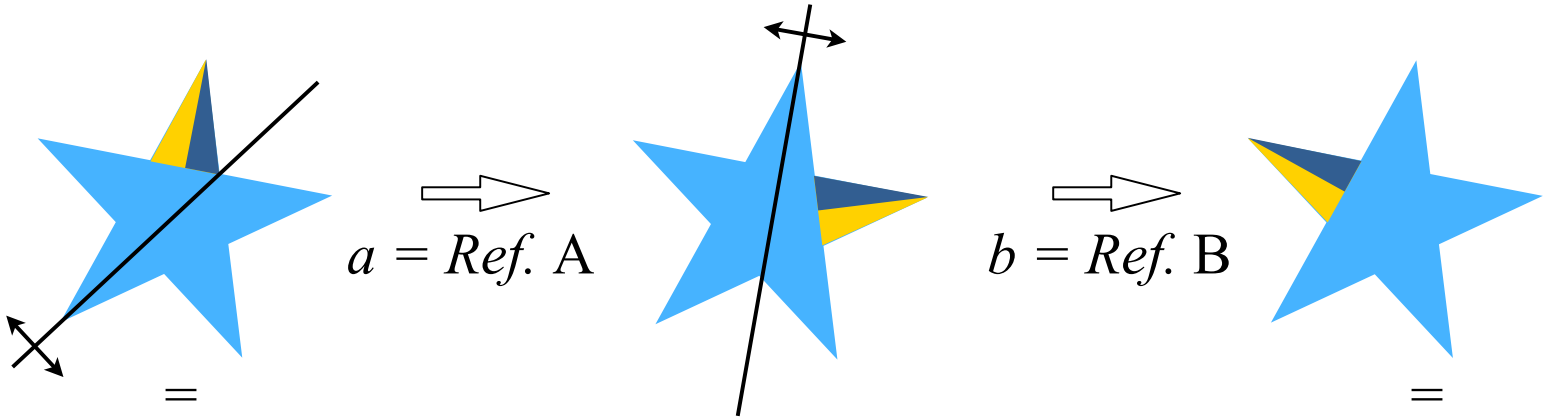
Symmetry Groups

Group Axioms

Dihedral Group D_5

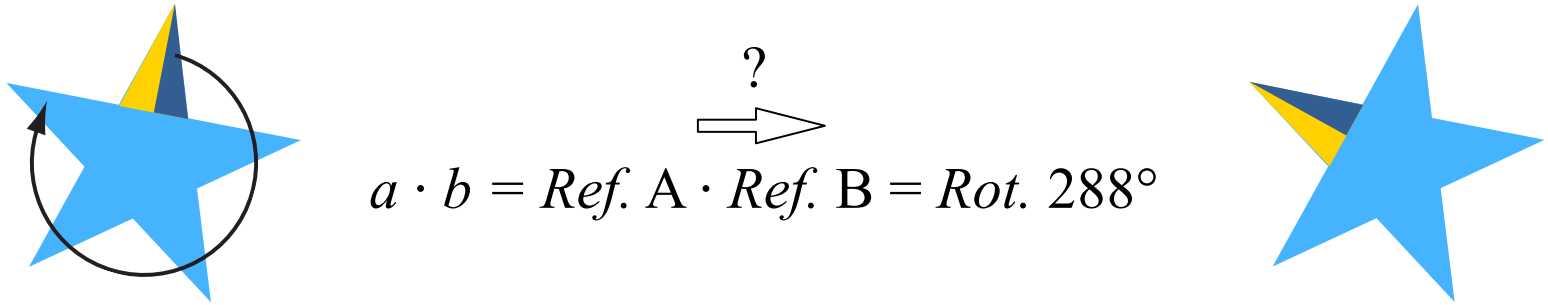


- *Closure* $a, b \in G \rightarrow a \cdot b \in G$



?

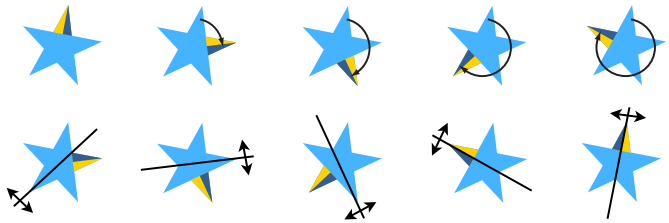
$a \cdot b = Ref.\ \mathrm{A} \cdot Ref.\ \mathrm{B} = Rot.\ 288°$



Symmetry Groups

Group Axioms

Dihedral Group D_5

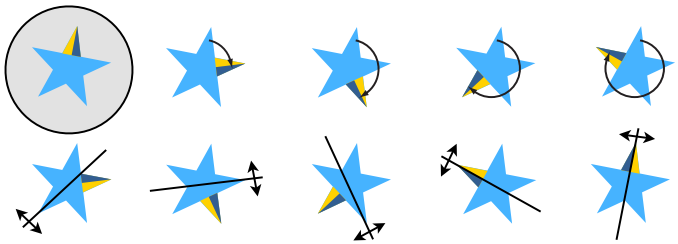


- *Closure* $a, b \in G \rightarrow a \cdot b \in G$
- *Associative* $a, b, c \in G \rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$

Symmetry Groups

Group Axioms

Dihedral Group D_5

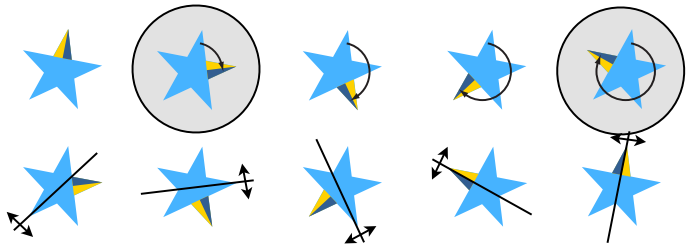


- *Closure* $a, b \in G \rightarrow a \cdot b \in G$
- *Associative* $a, b, c \in G \rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$
- *Identity* $\exists\, 1 \in G \rightarrow \forall a \in G : 1 \cdot a = a \cdot 1 = a$

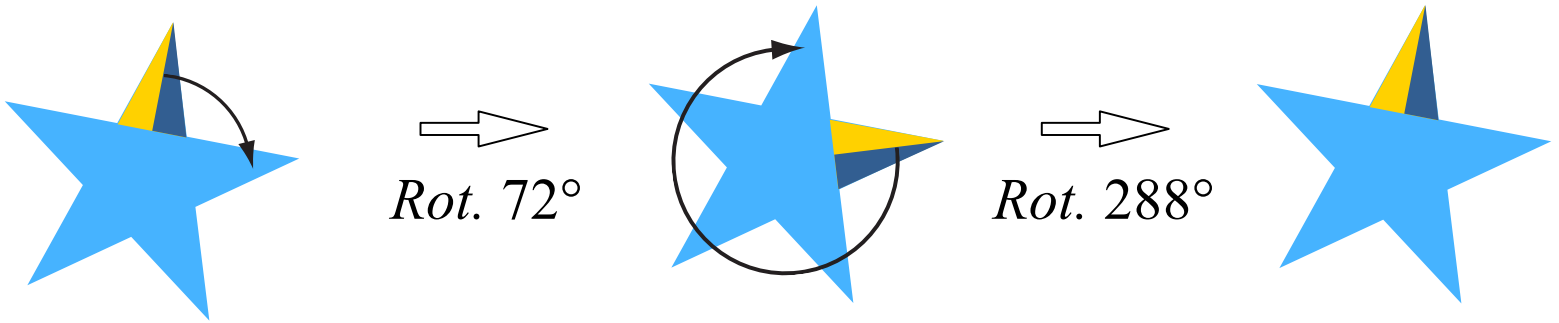
Symmetry Groups

Group Axioms

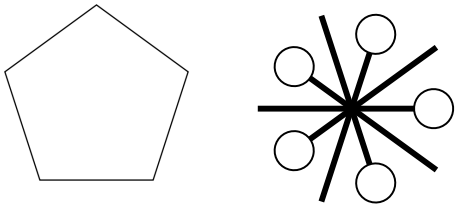
Dihedral Group D_5



- *Closure* $a, b \in G \rightarrow a \cdot b \in G$
- *Associative* $a, b, c \in G \rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$
- *Identity* $\exists\, 1 \in G \rightarrow \forall a \in G : 1 \cdot a = a \cdot 1 = a$
- *Inverse* $\forall a \in G\ \exists b \rightarrow a \cdot b = b \cdot a = 1$

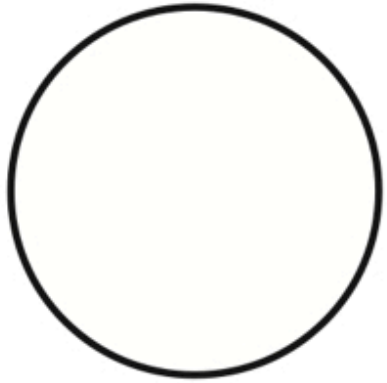


Symmetry Groups



dihedral group D_5

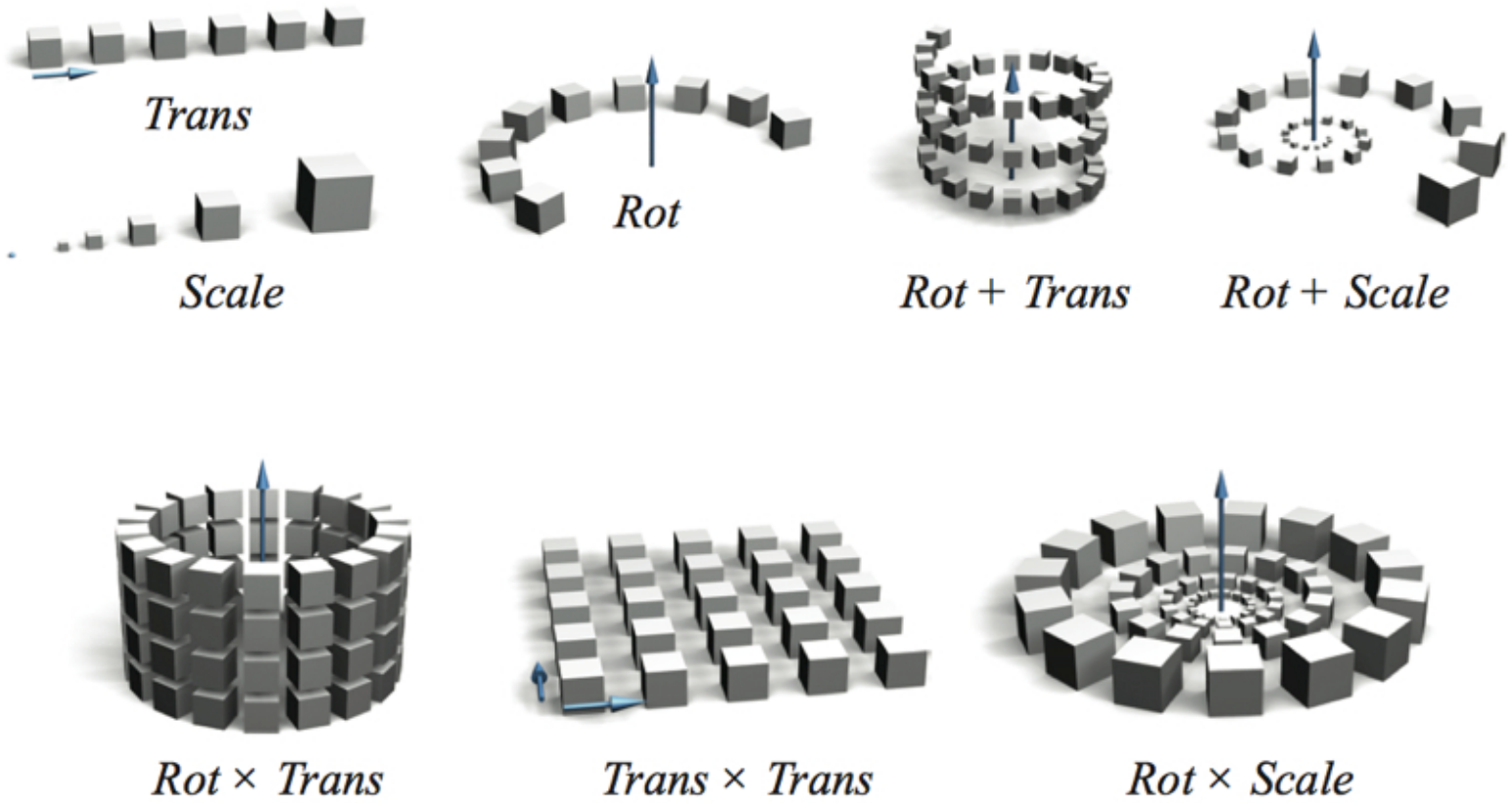
cyclic group C_3



infinite group $O(2)$

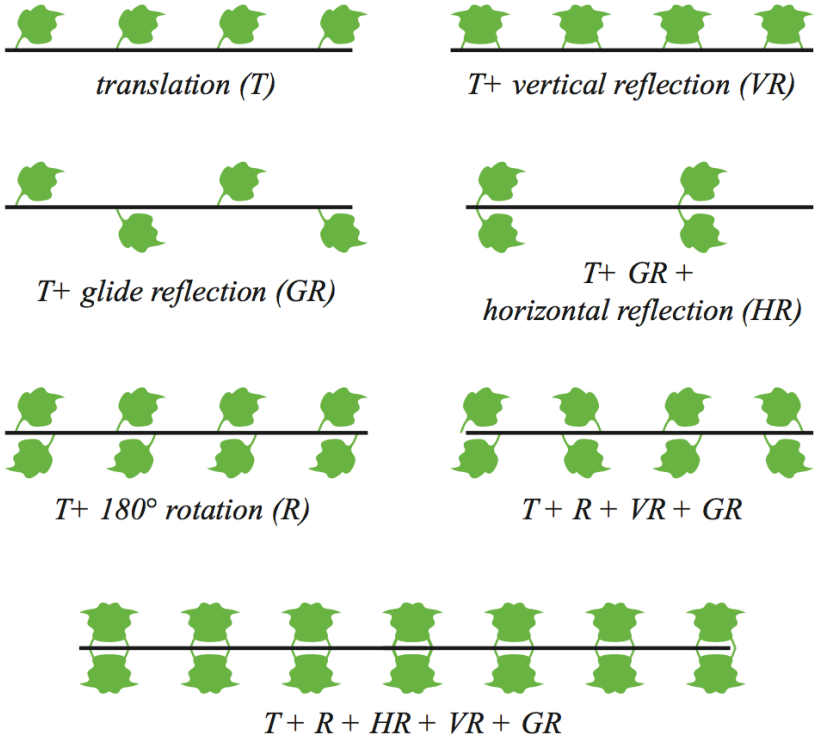
Symmetry Groups

Group Generators



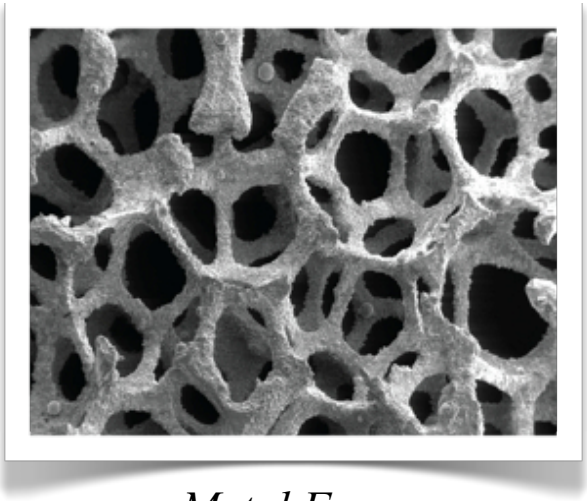
Patterns

1D - Frieze Groups

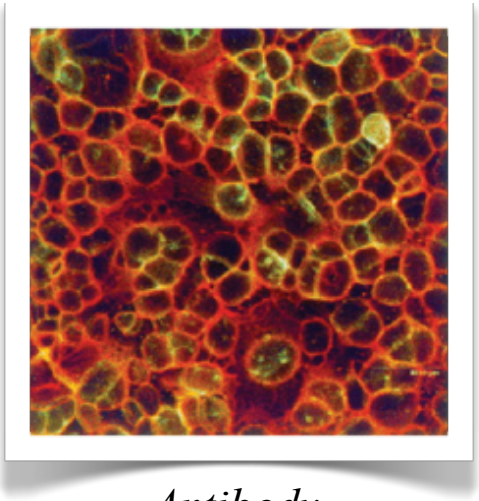


2D - Wallpaper Groups

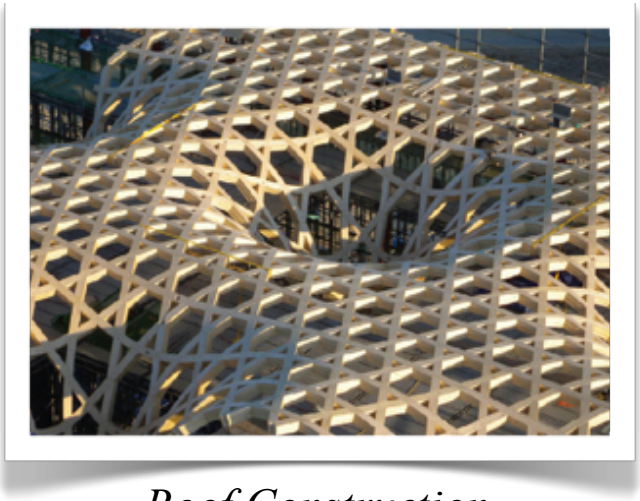
Symmetry Groups?



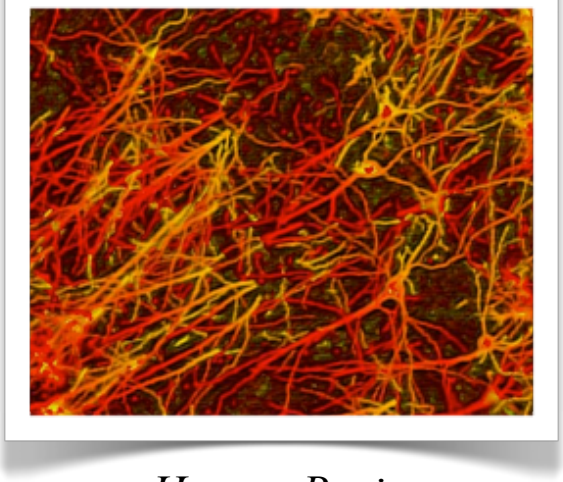
Metal Foam



Antibody

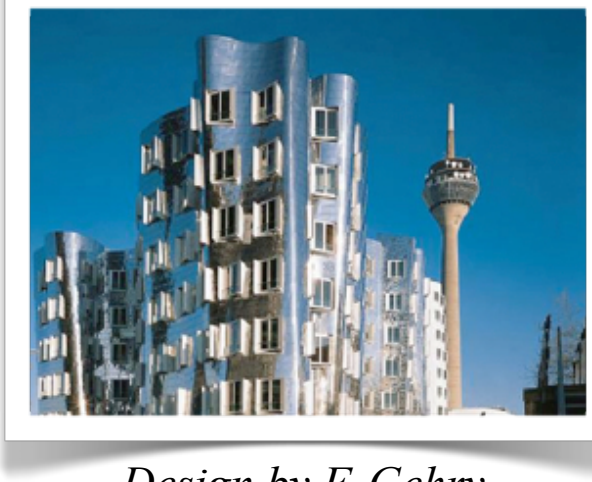


Roof Construction



Human Brain

Spiral Galaxy



Design by F. Gehry

Classification

Global vs. Partial

(a) complete symmetry group on parts of a shape

(b) partial translational symmetry

(c) partial rotational symmetry

Classification

Global vs. Partial

Exact vs. Approximate

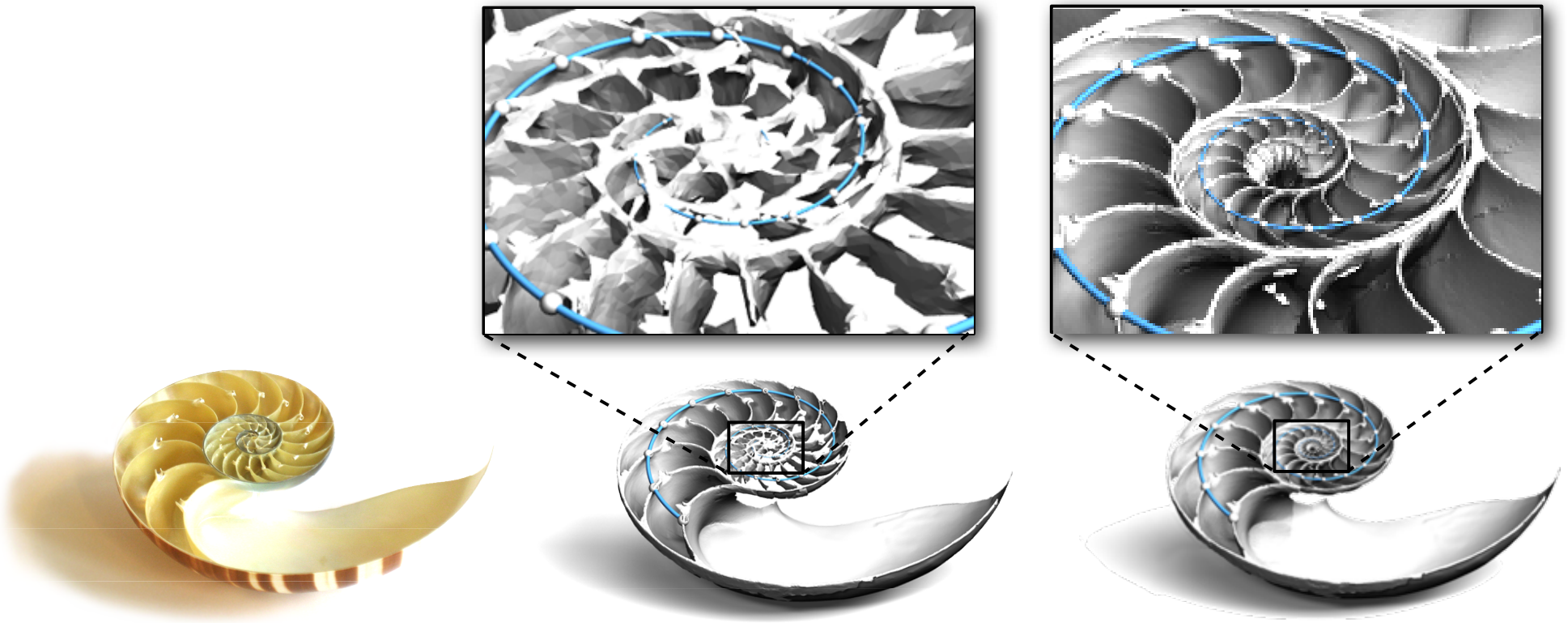
Classification

Global vs. Partial

Exact vs. Approximate

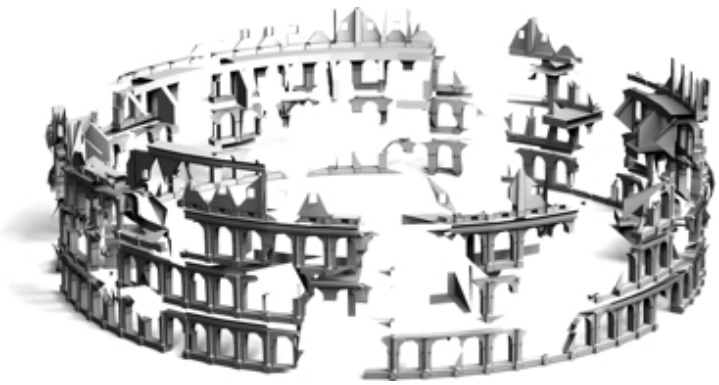
Intrinsic vs. Extrinsic

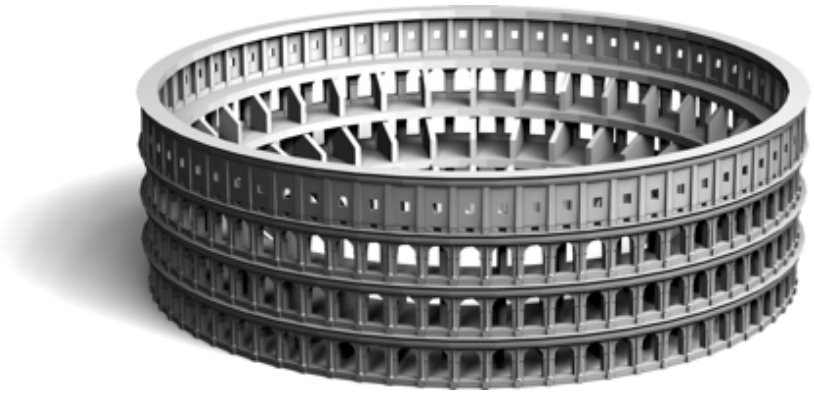
Understanding Geometry



Understanding Geometry

Symmetry encodes Redundancy

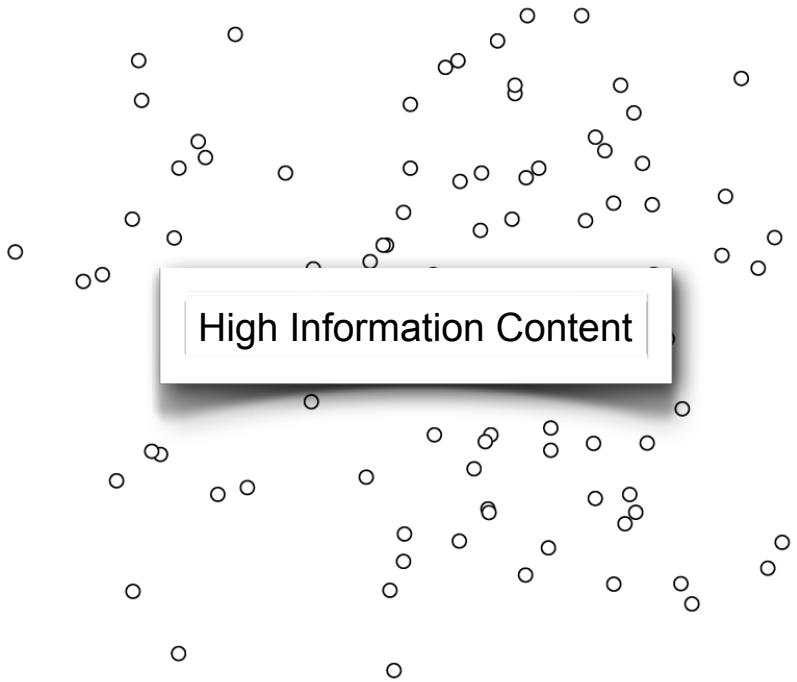




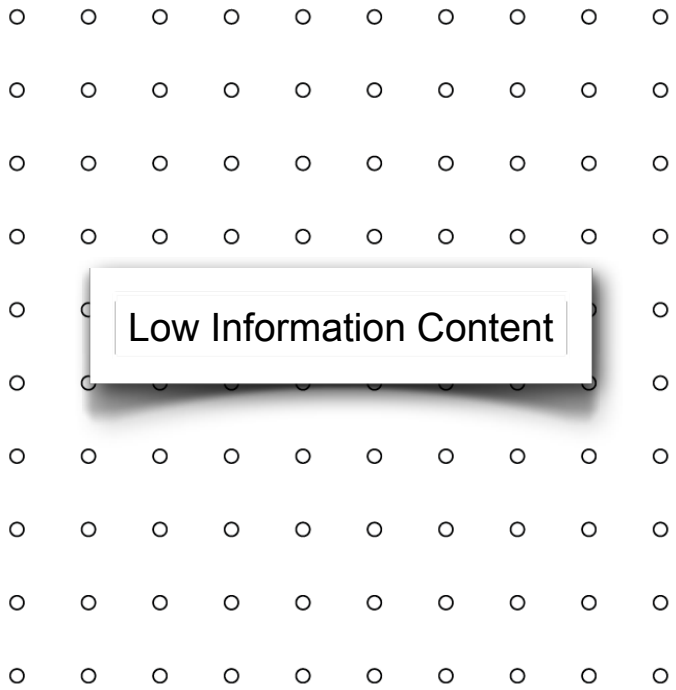
Symmetry Analysis → *Reconstruction*

Symmetry & Information

Symmetry is **Absence** of information



"100 Random Points"

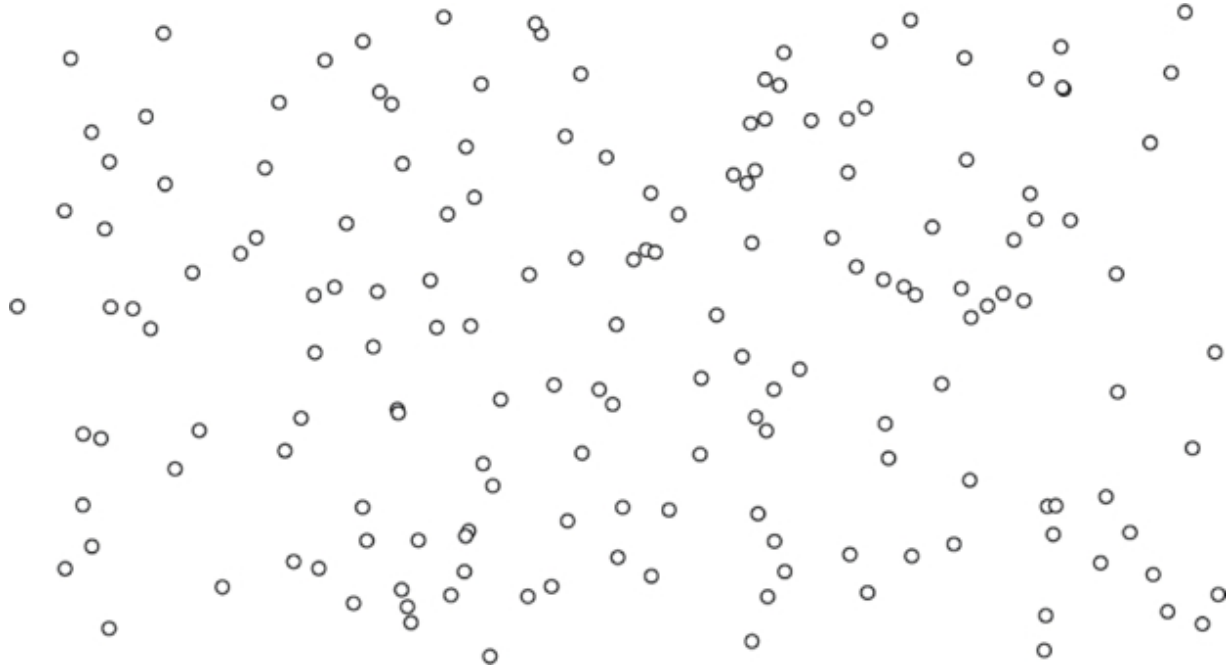


"A 10x10 Regular Grid of Points"

Symmetry & Information

Symmetry is **Absence** of information

→ structure discovery by **minimizing** representation cost



Symmetry & Information

Symmetry is **Absence** of information

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