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Folding Avoidance in Skeletal Strokes

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Abstract

Skeletal strokes is a method for mapping vector artwork defined along a straight path onto an arbitrary destination path. It is commonly used to give sketches a hand-drawn look, by mapping a scanned natural-medium stroke onto a user-drawn path. When the destination path turns corners or has areas with a small radius of curvature, the mapped artwork takes on an undesirable folded or creased appearance inside the corner or curve. We present a method to eliminate the folds in many cases by adjusting the skeleton before using it to map the artwork. While this cannot address all cases with folding, it gives greatly improved results in the cases that it can address.

Categories and Subject Descriptors (according to ACM CCS): I.3.4 [Computer Graphics]: Graphics Utilities—Graphics editors; I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Geometric algorithms; I.3.6 [Computer Graphics]: Methodology and Techniques—Interaction techniques;

1. Introduction

Skeletal strokes is a method for mapping vector artwork defined along a straight path to an arbitrary curved path. The basic approach was first defined by Barr [BAR84] but it was specifically described as a method for stroking paths by Hsu et al [HLW93; HL94]. Since then the functionality has appeared in vector drawing applications like Microsoft Expression Design (initially in its predecessor, Fractal Design Expression, in 1996) [FRO08] and Adobe Illustrator (initially in version 8, in 1998) [ADO98; ADO08]. One common use is to give sketches a hand-drawn appearance by applying scanned natural-media strokes to the sketched paths. Figure 1shows a vector sketch of a bird with three different skeletal stroke treatments, giving a variety of different appearances.

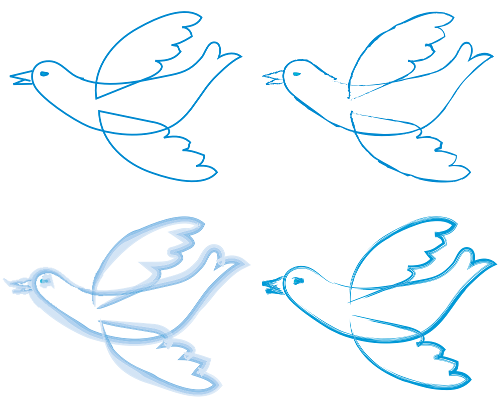

Figure 1: *A sketch of a bird with three different skeletal stroke treatments.*

The basic technology for skeletal strokes is straightforward, as shown in Figure 2. Because the artwork associated with natural media strokes is usually complex, most examples in this paper use simpler art that more clearly shows the techniques involved. The skeletal stroke algorithm begins with a piece of vector artwork, the prototype, and a destination path to which the stroke is applied. The destination path is treated as a spine, and ribs are constructed perpendicular to the path. Each rib is as long as the prototype artwork is wide. Corresponding ribs are constructed in the prototype artwork, spaced proportionally to the ribs on the destination path. The places where the vector artwork crosses the ribs in the prototype are mapped to locations on the ribs along the destination, and these mapped locations are used to construct new paths that have been mapped to the destination.

Figure 3 shows that problems occur where the radius of curvature of the destination path is smaller than the half-width of the prototype artwork. In this case, the ribs cross each other, and the resulting stroke exhibits places where the artwork folds back upon itself. In particular, these problems

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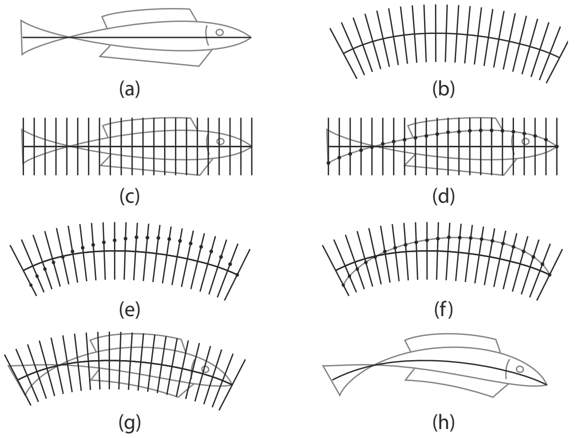

Figure 2: *The basic skeletal stroke algorithm. (a) The prototype artwork. (b) The destination path, with ribs. (c) Ribs applied to the prototype. (d) A single path's intersection with the ribs. (e) The intersections mapped to the destination skeleton. (f) The resulting path. (g) Repeated for all prototype paths. (h) Without the ribs.*

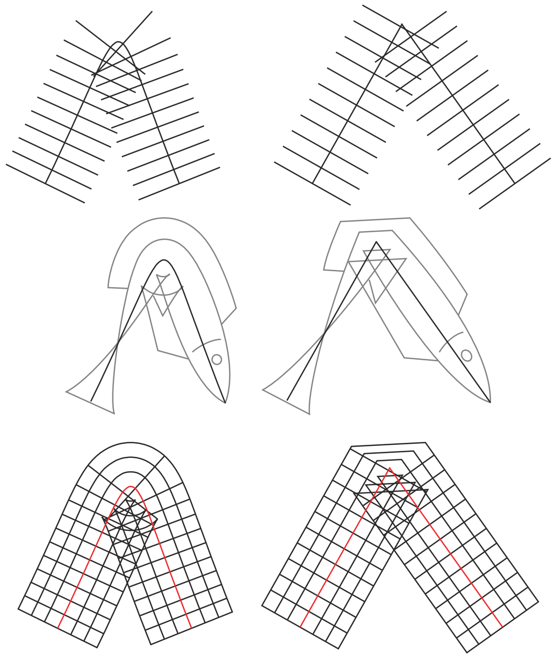

Figure 3: *Skeletons and the resulting folding when the destination path makes a sharp curve or a corner. Although a grid for the prototype artwork would be uncommon in real artwork, it shows the effects of folding very clearly.*

occur wherever the destination path has a corner, since the radius of curvature at the corner is zero.

In this paper we introduce a new technique that improves the appearance of the mapped artwork in these problem areas. Figure 4 shows the bird sketch with a watercolor

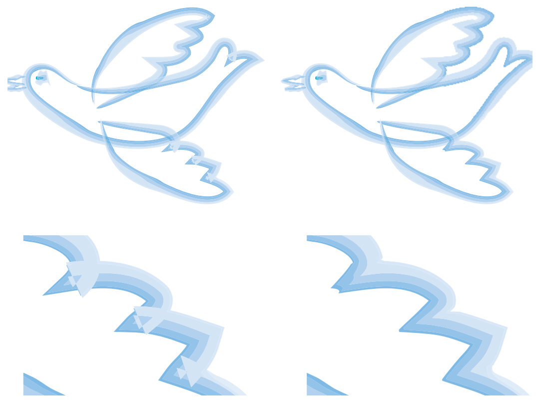

Figure 4: *A watercolor stroke applied to the bird sketch without and with our folding elimination technique. The lower illustrations show a close-up of the lower wing.*

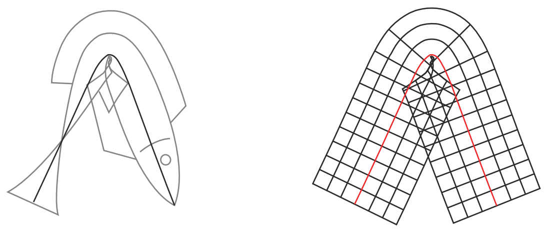

Figure 5: *Adjusting folded areas by limiting artwork to the radius of curvature of the destination path.*

stroke applied without and with our folding elimination techniques

2. Previous Work

Hsu et al [HLW93] describe two techniques for improving the appearance of folds, one applying to curved paths, and one applying to paths composed of straight segments (polylines).

For curved paths, they adjust the mapping of points on the destination ribs so that they are never further from the destination path than the path's radius of curvature at the rib's construction point. This has the effect of compressing the prototype artwork down to the locus of the radius of curvature of the path. Figure 5 shows how this leads to somewhat diamond-shaped folds rather than the somewhat triangular folds that occur with no adjustment. The problem with this technique is that it does not eliminate folding completely, and it is not clearly superior to doing no adjustment at all.

Hsu proposes a different adjustment technique for polylines. In this case the ribs at corners are adjusted to follow the angle bisectors at the corner points, while ribs at path ends remain perpendicular to the path. Intermediate ribs interpolate between these, tilting towards the point where

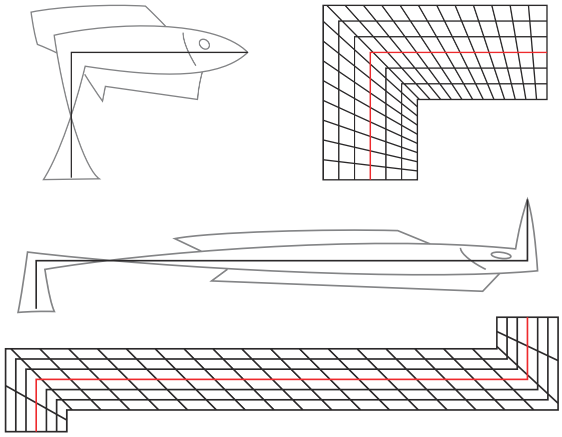

Figure 6: *Adjusting folded areas on polylines by making corner ribs follow the angle bisectors.*

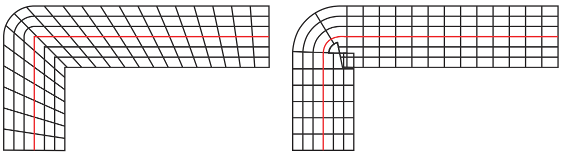

Figure 7: *Different results from the corner and curve adjustment algorithms.*

the ribs at the adjacent vertices would intersect if extended. This has the effect of eliminating folding in the corners and distributing the adjustment smoothly between vertices. Figure 6 shows the result of applying this technique. Figure 7, on the left, shows a variant that produces rounded corners.

There are several problems with this technique. One is that it is completely different from the curve technique. Two paths that differ only slightly, with one having a corner and the other a small curve, give quite different results as shown in Figure 7. By distributing the distortion along the full length of a segment, it causes adjustment effects to extend far from the corners, leading to an overall skewed appearance as shown in Figure 6.

There is also a curious omission in the description of the two techniques. The first applies to curved areas on curved paths, and the second to corners on polyline paths. There is no technique for corners on curved paths. The first technique has no effect when applied to a corner, and the second relies on using the intersection of the extended vertex ribs, which only gives meaningful results when the path contains only straight line segments.

3. Eliminating Folds

A concrete definition of folding is necessary in order to have a clear definition for what it means to eliminate folds. Let us choose two ribs in the prototype artwork, adjacent or not, and isolate the portion of the prototype between these ribs. We can consider this portion as a planar graph of intersecting paths. A mapping of this portion of the prototype onto a destination path is unfolded if the result of the mapping, considered as a planar graph, is isomorphic to the original portion. The mapping is folded if the graph is not isomorphic. It either has path intersections that do not exist in the original portion of the prototype, or it is missing some intersections that do exist in the original portion. In Figure 8, the left and right mappings are not isomorphic to the original grid prototype, while the center mapping is.

This definition is a bit too strict to be useful. Instead, we generalize it slightly and allow two vertices in the mapped graph to be collapsed into a single vertex, as long as these vertices are along the outside edge of the prototype. We will call this a loosely unfolded mapping. The right mapping in Figure 8 shows the difference.

An unfolded mapping of the entire prototype is not always desirable. The destination path can intersect itself, or separate portions of it can approach each other without intersecting. These both lead to intersections in the mapped artwork that are probably desirable, as shown in Figure 9.

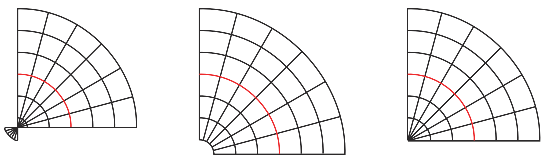

Figure 8: *A folded, strictly unfolded, and loosely unfolded mapping of a grid prototype onto a quarter circle.*

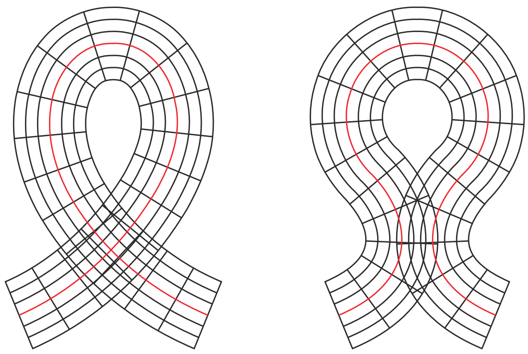

Figure 9: *Folded stroke applications that are not objectionable.*

We assert that the standard skeletal stroke algorithm always gives a strictly unfolded mapping along a section of the destination path if no rib in that section intersects any other rib in that section. It gives a loosely unfolded mapping if adjacent ribs are allowed to intersect each other only at their endpoints. The converse is not necessarily true for all prototypes; crossing ribs can always potentially give rise to a folded mapping (the prototype could be identical to the ribs, so if the ribs cross, so do segments in the mapped art), but the actual prototype may not cause it to occur. Figure 10 shows a prototype in black with ribs in gray, and an application that has crossing ribs but an unfolded mapping. We are not considering artwork-specific mapping algorithms, so we consider a section of the skeleton problematic for folding whenever there is some prototype that could give a folded mapping.

We would like to eliminate folds when possible while maintaining the following constraints:

- The spine of the prototype must be mapped onto the destination path.
- The envelope of the resulting skeletal stroke must be such that using a solid filled rectangle with width *w* as the prototype gives a result that is indistinguishable from a solid filled stroke of width *w*, as generated by standard stroking algorithms.
- Portions of the skeletal stroke that are not in an area requiring folding adjustment appear as they would if they were part of a path that did not need adjustment anywhere. In other words, the adjustment must be limited to areas with folding problems.

There are two steps to eliminating folds. First, we identify areas of the path that require adjustment, and then we adjust the ribs in these areas so that they intersect only at their endpoints. Applying the skeletal stroke algorithm to the resulting skeleton gives rise to a loosely unfolded mapping.

3.1. The Unadjusted Stroke Envelope

An important construct for our algorithm is the unadjusted stroke envelope. Conceptually, create a line segment that is as long as the prototype is wide, and sweep it along the destination path, remaining perpendicular to the destination path. The unadjusted stroke envelope is the pair of paths that are traced by the two end points of this segment. Concretely, this is the same as the two paths that result from applying the skeletal stroke algorithm with no folding adjustment to a prototype consisting of a pair of line segments that bound the sides of the actual prototype. The two paths in the envelope pass through the end points of the destination ribs, in order, along the path. Figure 11 shows the unadjusted stroke envelope for the fish prototype when applied to a tightly curved path.

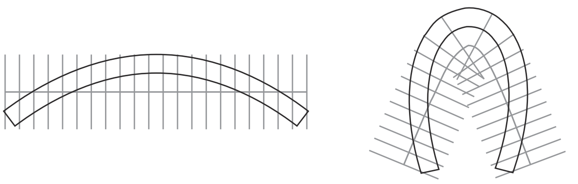

Figure 10: *A mapping that could give rise to folding, but doesn't for the particular prototype.*

We also must define what the unadjusted stroke envelope does on the outside of corners. There are several options, corresponding to different notions of corners. Consider two points on the envelope infinitesimally located on either side of the corner. A bevelled corner results from connecting these directly with a line segment. A rounded corner results from connecting them with a circular arc centered on the corner point of the destination path. A mitered corner results from extending them with two segments tangent to the envelope path. Figure 12 shows envelopes for these three corner treatments, but other treatments are also possible. Our method works with any treatment that creates a convex corner area.

3.2. Identifying Adjustment Areas

The paths in the unadjusted stroke envelope display retrograde motion wherever the radius of curvature of the destination path is smaller than the half width of the prototype.

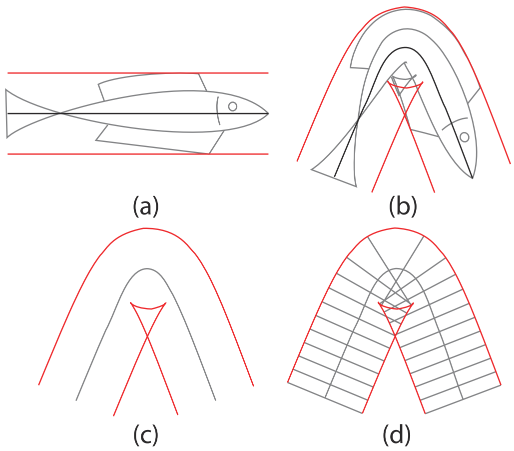

Figure 11: *The unadjusted stroke envelope: (a) A prototype with bounding lines. (b) Applied to a curved path. (c) The unadjusted stroke envelope. (d) How the envelope connects the ends of the ribs.*

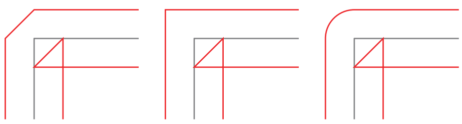

Figure 12: *Bevelled, mitered, and rounded corners in the unadjusted stroke envelope.*

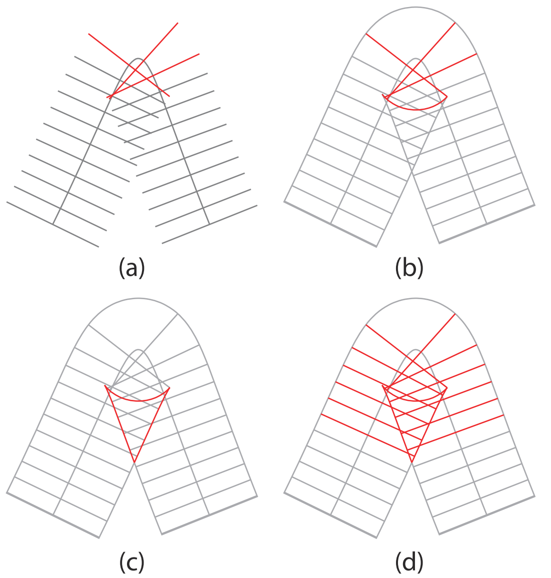

Figure 13: *The steps in finding the adjustment area.*

The first step in identifying adjustment areas is identifying these retrograde sections. This is straightforward because the calculation of the radius of curvature is a by-product of rib construction. Alternatively, these areas can be identified as areas along the destination skeleton where a rib crosses an adjacent rib, one that is immediately before or after. Figure 13 (a) shows the ribs in the retrograde area in red.

The retrograde portion of the unadjusted stroke envelope is the section that connects the ribs in this tightly curved area, as shown in Figure 13 (b). For a destination path with a corner, the retrograde portion is the section that connects the two ribs on either side of the corner

Hsu's curved path adjustment [HLW93] limits the adjustment to areas along the destination path that give rise to retrograde portions of the stroke envelope. To improve upon these results, we identify adjustment areas that extend for longer stretches of the path.

To extend the adjustment area, we consider the two sections of the unadjusted stroke envelope on either side of the retrograde portion and use standard curve intersection techniques to search for the point where they first cross, as shown in Figure 13 (c). If we find such a point, then the area to be adjusted consists of the set of ribs that end on the segment of the unadjusted stroke envelope between where it intersects itself, as shown in Figure 13 (d). We call the intersection point the *center of crossing*.

3.3. Adjusting the Skeleton

The adjustment process is also straightforward. Each rib in the adjustment area is modified. One end is placed at the center of crossing. The rib then continues through the same point on the destination curve as the original rib, and it ends on the outside stroke envelope path. Figure 14 shows the details of one adjusted rib, with the original unadjusted rib shown in black and the adjusted rib shown in red. Figure 15 shows completely adjusted skeletons for a curved and corner case, as well as the result when used on the fish prototype.

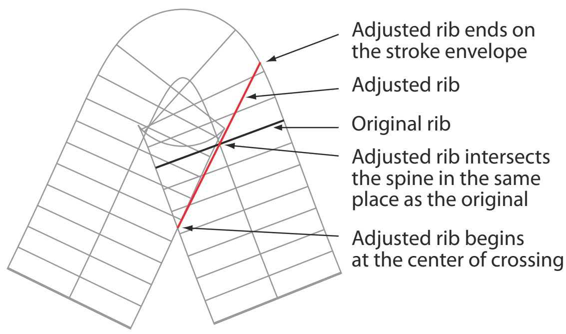

Figure 14: *Adjustment for one rib.*

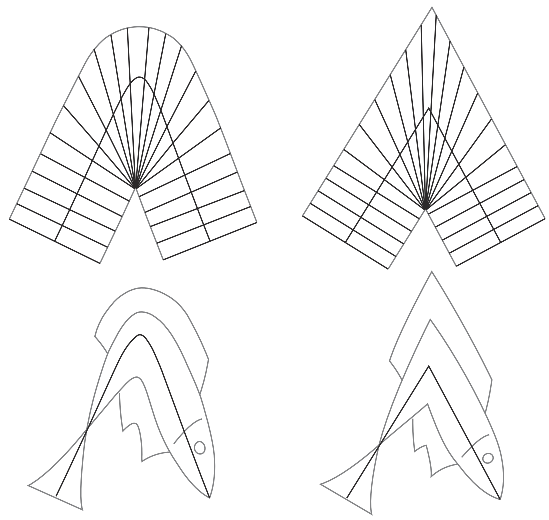

Figure 15: *Adjusted skeletons and the resulting stroke.*

To achieve the goal of having the center line of the prototype follow the destination path, we slightly modify the skeletal stroke algorithm to treat a rib as two pieces that do not necessarily have the same lengths. In the curved skeleton example, the portion of the rib inside the curve is longer than the portion outside.

These adjusted ribs differ from the original ribs in two ways. They are no longer perpendicular to the destination path, and they are no longer exactly as long as the source artwork is wide. However, they are all segments of rays that initiate at the center of crossing, so they do not intersect each other anywhere but at the center of crossing. Because the center of crossing lies along the outside edge of the stroke, a mapping done with these ribs will be a loosely unfolded mapping.

To achieve the third goal of adjustment, limiting the adjustment to areas that need it, we add new perpendicular ribs to the skeleton at the points of the destination curve that correspond to the center of crossing. This is especially important for destination paths that have straight lines. The standard method for constructing the original ribs generally spaces them more distantly where the path has high radius of curvature; in specific, places where the path is a straight segment typically have no internal ribs. These extra ribs prevent the skewed appearance that results from Hsu et al's adjustment algorithm [HLW93].

3.4. Double Adjustment

There are cases where the destination path makes a compound bend like an s-curve that would require adjustment on both sides. In this case there are ribs that lie within both adjustment areas, and could be adjusted using either side's center of crossing. Here we construct a segment connecting the two centers of crossing and find where it crosses the destination path. Ribs are adjusted according to which side of this crossing they occur on. Figure 16 shows an example of this with the rib on one side of the crossing shown in red and the ribs on the other side shown in blue.

4. Evaluation, Limitations, and Future Work

We now compare these results to the results without adjustment and with the adjustments described by Hsu et al [HLW93].

Aesthetics: Although this is unquestionably a subjective area, a collection of experienced Illustrator users unanimously preferred the results of our technique to the unadjusted results or the results with Hsu's method. Figure 17 shows a natural medium brush stroke applied to the same path with no adjustment, with Hsu's adjustment method, and with our method. Figure 18 shows a more elaborate illustration that uses a variety of brush strokes. Since this is an aesthetic choice, there is no reason an application could not give users a choice in case they prefer another result.

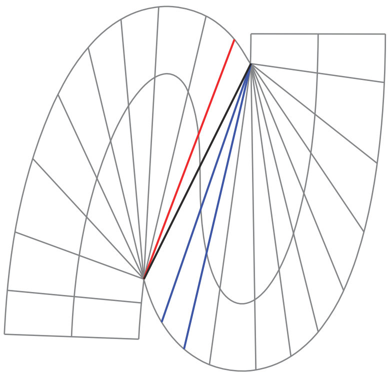

Figure 16: *A case where ribs could be adjusted in multiple ways.*

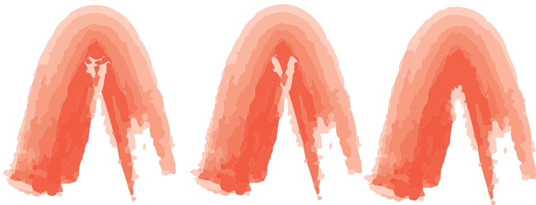

Figure 17: *A skeletal brush stroke with no folding adjustment, with Hsu's adjustment method, and with our method.*

Figure 18: *Brush strokes in an actual illustration, without and with folding adjustment.*

Generality: Hsu's method for adjusting inside curves does not apply to corners in curved paths, and the adjustment method for corners applies only to polylines. Our method makes no distinction between corners and curves or between curved paths or polylines. It can be applied in more cases, and produces results that are very similar when corners are replaced by curves.

Locality: The addition of extra spines limits the adjustment effect to areas that need adjustment, avoiding a skewed appearance in other areas of the destination path.

While our technique improves folding in many cases, we unfortunately cannot completely eliminate folding because there are cases where our method cannot be applied.

The most common case involves being unable to find the center of crossing. If a folding problem area occurs near the end of the destination path, the stroke envelope can end before it crosses itself. We also must exclude closed paths where the stroke envelope connects to itself before it intersects itself. If the search for the intersection point reaches the ends of the path without finding an intersection, or if it

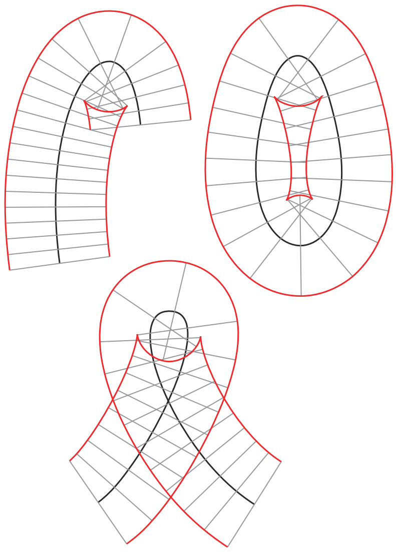

Figure 19: *Skeletons that cannot be adjusted.*

reaches the same point on the path in coming from either direction, the technique cannot be applied. We must also exclude cases where the destination curve crosses itself within the identified adjustment area. Figure 19 shows three skeletons that cannot be adjusted because the envelope does not intersect itself.

There can also be cases where the destination path has small details in an adjustment area, which can lead to the adjusted ribs not being strictly in clockwise or counterclockwise order around the center of crossing as they connect to points on the path. These cases often result in a folded mapping

At the moment, encountering the situations above requires either doing no adjustment, or falling back on the Hsu's adjustment method. We would prefer to do better.

Some completely new adjustment technique is needed when our method fails, either because it cannot find a center of crossing or because the ribs are not in order. We believe that a method based upon preventing the mapped artwork from crossing the medial axis of the area inside the curve could give good results, but we have not yet implemented it.

5. Conclusions

Figure 20 compares our results to unadjusted results and Hsu's results. It shows several prototypes applied to polyline paths with a corner, curved paths with a tight curve, and curved paths with a corner. Our results eliminate the folding in all of these cases and avoid skewing the unadjusted artwork in the polyline case.

It is difficult to precisely quantify how frequently our improvement can be applied, and how frequently it cannot. The problem cases are not exactly uncommon, but experience suggests that they are much less common than cases where folding can be eliminated. Our polled users all indicated that this would be a welcome improvement even though it does not address every case of folding.

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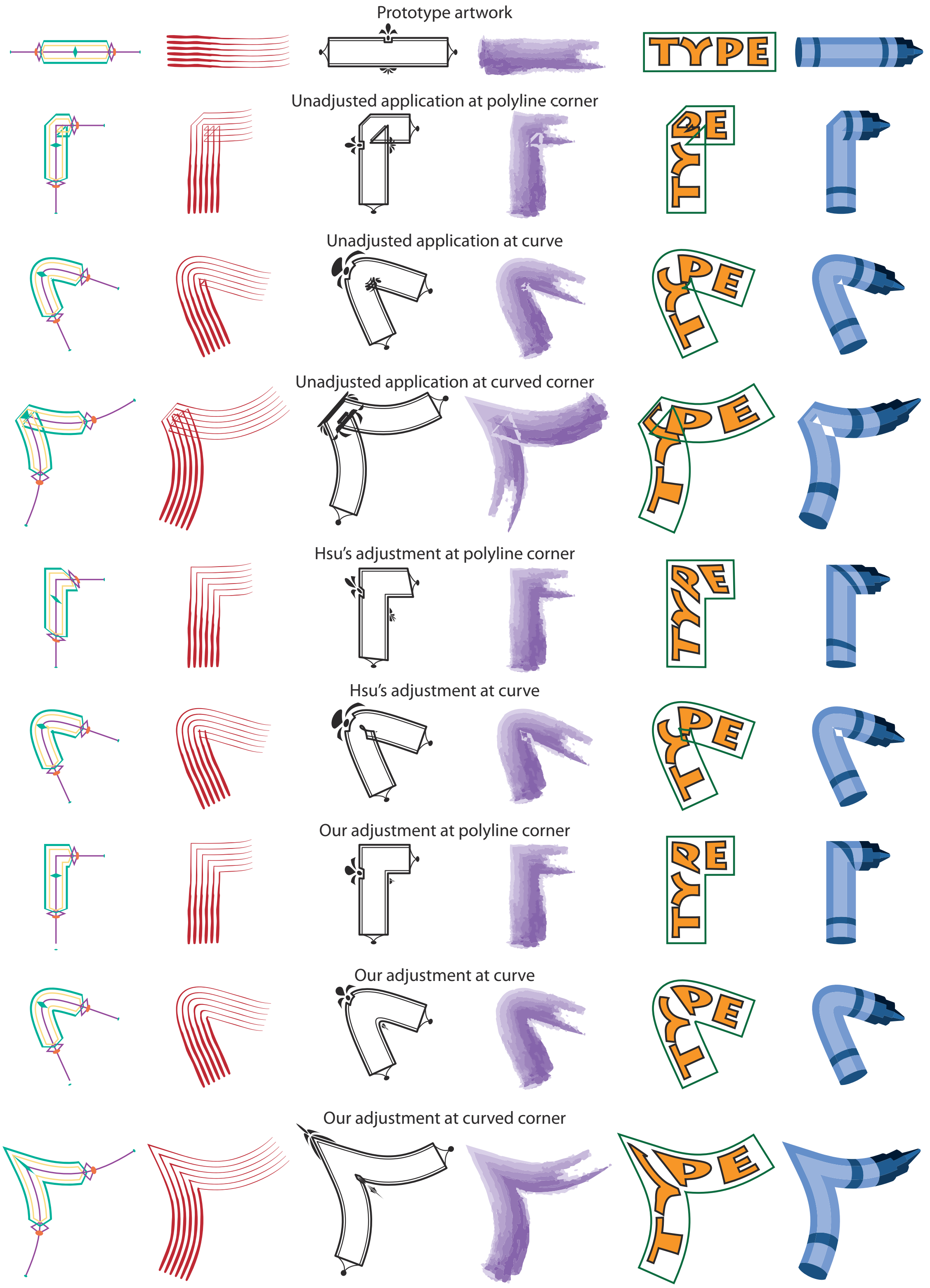

Figure 20: *A variety of strokes applied to a variety of paths. The paths follow the center line of the leftmost examples.*